Free and Open Systems Theory

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Abstract

The focus of system theory to date is mainly local, developing the complex self-organising characteristics of *intra*connection but with *inter*connection to an independent environment. Now with the increasing importance of interoperation of global systems the spotlight has to be broadened on to non-local activities in system theory and the role of systems that are not just open but also free. Non-locality requires closer attention to fundamental definitions which can no longer rely on local assumptions found currently in General Information Theory. Category theory recommended by Robert Rosen as a modern tool for living systems is found to have a formal expressive power as process beyond modelling for exploring the fundamental non-local concept of adjointness needed to understand advanced systems.

1 Introduction

The pursuit of wealth to satisfy the needs of the day has to be expressed in today's currency. The same is true for the wealth of ideas including the great principles of systems theory. In our work we are concerned with interoperating information systems but these are just instances of the much wider need for systems theory to keep up with our current technological age. Much today is concerned with globalisation whether technical, economic, political or social and for systems theory it is the concept of a free and open system that is paramount for globalisation. Information systems share many of the characteristics and needs of other modern and postmodern network science [Watts and Strogatz 1998] which enshrines a concept of freeness and openness such as life, consciousness and intelligence, biology and medicine [Klir 1993], quantum phenomena in nanotechnology, etc. The large global systems needing much improved understanding include pandemics, prediction of earthquakes, world energy management policy and climate change but it is the same characteristics of freeness and openness that are needed for narrower but complex research like embryonic stem cell research, microtubule dynamics [Gr-

ishchuk *et al* 2005], chaotic carriers for broadband [Argyris *et al* 2005], pollution control, hazardous substance tracking, genetically-modified crop seed propagation and other biological engineering [Endy 2005], etc. These are of immediate significance but other research of long-term vital importance to the human race is how to harness sufficient power to bring us up to a Type I civilization able to control geophysical forces as in Kardashev's classification [Kardashev 1964]. No doubt in due course advanced systems theory will be needed to attain the higher Types II and III civilization [1]. The pervasive theme is one of naturalness.

Natural entities are always easier to recognise than to define and the notion of a system is notoriously difficult. There is a reluctance in the literature for writers to attempt any kind of definition – even an informal one. There is certainly a dearth of formal definitions whether in words or symbols of the nature of a system as a whole. The interest usually is in defining in informal terms various advanced system components like feedback and special attractors. However fully rigorous definitions are needed if progress is to be made in understanding global systems.

Second-order cybernetics makes explicit what was implicit in the work of the founding fathers that the observer is part of the system. The rider to this is that components modelling the system should be distinguishable from the components of the system itself [Heylighen and Joslyn 2001]. These issues bring to the fore the question of uncertainty which has led to the development of General Information Theory (GIT) drawing on facets of AI, databases, neural nets and fuzzy set theory but also recognising the need for a constructive approach [Klir 2003].

The basis for this development was still classical relying on methods derived fundamentally from Shannon's theory of information. That is limited to intensional and syntactical nature of information and not (as Shannon regretted) to meaning. As it is also set based, it is also limited to local conditions. Natural systems however and global systems are non-local. The classical cybernetics or system paradigm deals

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[1]Already there are signs of this in risk management for asteroid collision with the design of gravitational tractors for towing asteroids [Lu and Love 2005].

with a system in an environment. With global systems the environment is itself a system of interacting systems. Problems like Russell's paradox immediately appear. A solution proposed by Robert Rosen (who perhaps in his lifetime progressed further than anyone else in formally representing the nature of life and living systems) was to recommend a shift from set theory to category theory - - "the natural habitat for discussing . . . specific modelling relations" ([Rosen 1991] p.153). We are pursuing here Rosen's prescription and find that it is more perceptive than his comments at first sight suggest. Category theory may go further in resolving the questions raised in second-order cybernetics and the problem of the model itself.

1.1 Connectivity and Activity

The basic concept recognised for a system is normally the internal connectivity of the components of the system. The concept of a complex whole consisting of intraconnecting parts goes back at least to the classical Greek word *systēma* which (although more common in later Greek) is found occasionally in the work of the early philosophers. Plato uses the word to describe a government institution and Aristotle applied it to a literary composition. Intraconnectivity is geometric and difficult to formalise in algebraic terms. By its axioms set membership consists of independent elements and therefore a system cannot be conveniently represented in terms of first-order set theory.

The biologist and founder of modern systems theory Ludwig von Bertalanffy [2] sought to refute the argument that 'the definition of systems as sets of elements standing in interrelation is so general and vague that not much can be learned from it' ([Bertalanffy 1968] p.37) by putting forward the formalisation of a system as a finite sequence of differential equations

$$\frac{dQ_i}{dt} = f_i(Q_1, Q_2, \ldots, Q_i)$$

for some quantity Q ([Bertalanffy 1968] p.55). An application example for an open system is cited below.

In this he ([Bertalanffy 1968] p.86) seems inspired by the 'new ontology' of Nikolai Hartmann as a theory of categories to be replaced 'by an exact system of logico-mathematical laws. General notions as yet expressed in the vernacular would acquire the unambiguous and exact expression possible only in mathematical language'. However it is only now with the development of formal category theory that the power of this replacement of the vernacular with the mathematical can be fully realised. It can also make patent many latent assumptions in the theory of differential equations.

A system is to be treated as a complex structure as for instance in Peter Checkland's definition:

A system is a model of a whole entity; when applied to human activity, the model is characterised fundamentally in terms of hierarchical structure, emergent properties, communication, and control ([Checkland 1981] p.318).

The major components of complexity are openness and freeness but the distinctive characteristic is 'natural activity' like self-organisation, a concept first introduced by William Ross Ashby [Ashby 1947] and still of great importance as intra-activity but now joined by the phenomena of anticipation [Klir 2002] and interactivity between systems to be found in global interoperability. We may classify systems from current interests as in the table of Figure 1. However, these terms have been found difficult to formalise.

The transition from connectivity to activity involves a type change and therefore requires a formal system with an inbuilt facility to cross between levels. Thus intraconnectivity between components cannot give rise to interactivity between those components without some non-local integrity coming into play. This is a related problem to that found in Russell's paradox which has yet to be resolved theoretically despite attempts by Spencer Brown with his *Laws of Form* and Russell with his type theory[3].

system	natural relationship	locality
closed	*intra*-connectivity	local
open	*inter*-connectivity	local
self-organised	*intra*-activity	non-local
free	*inter*-activity	non-local

Figure 1: Key Elements in the Definition of a System

1.2 Open System

The property of openness was early recognised [Bertalanffy 1950; Bertalanffy 1950b]. The simplest definition of an open system is one that can be accessed. ([Skyttner 1996] p.38) considered that an open system depends on an environment where it can exchange matter, energy and information whereas a closed system is open for input of energy only. The concept of *open* is normally defined inductively on the open interval. But this has taken some time to crystallise. For the last fifty years (([Jeffreys and Jeffreys 1956] preface to first edition) the 'Bible' reference text on the use of mathematics in the physical sciences has some difficulty with openness relying on example in lieu of rigorous derivation. Sir Harold and Lady Jeffreys ([Jeffreys and Jeffreys 1956] present it with some diffidence [4] as: 'A region is *closed* if all its boundary

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[2]According to von Bertalanffy ([Bertalanffy 1968] p.9) the term *system* is synonymous with 'natural philosophy' (Leibniz); 'coincidence of opposites' (Nicholas of Cusa); 'mystic medicine' (Paracelsus); dialectic (Hegel and Marx); physical *gestalten* (Köhler); organic mechanism (Whitehead).

[3]Russell himself described his own type theory as 'not really a theory but a stopgap' in the preface to the first American edition of ([Spencer Brown 1969] pp.xiii-xiv).

[4]Even switching the notation between their second and third editions but then unable to apply the notation consistently ([Jeffreys and Jeffreys 1956] p.19-20).

points are members of it, *open* if all it points are interior points'. However these concepts are bound up with the question of continuity.

The best that classical analysis can provide as a basis for openness is the Dedekind cut which is always available between nests of intervals in any dense field of numbers ([Jeffreys and Jeffreys 1956] pp.1-2) [5]. The critical weakness of Dedekind openness is that it is a section of a pre-defined field and a very poor basis therefore for properties of openness like emergence, creativity and non-locality. For the Dedekind is always local.

In topology it is a little clearer. [Kelley 1955] (pp.37-39) proves that a set is open if and only if it contains a neighbourhood of each of its points where a set U in the topological space $(X, \Im)$ is a neighbourhood of a point x if and only if U contains an open set to which x belongs. Every open set is a neighbourhood of each of its points. Each neighbourhood of a point contains an open neighbourhood of the point. A topology $\Im$ has the intersection of any two of its members as a member of $\Im$ as well as the union of the members of each subfamily of $\Im$. The members of the topology $\Im$ are called open relative to $\Im$, or $\Im$-open, or if only one topology is under consideration, simply open sets. The members of the topology $\Im$ are called open relative to $\Im$ or $\Im$- open ([Kelley 1955] p.37-39). Notice this relative topology. A system is open to its environment but in the same way it may be open to more than one environment. Furthermore a system is integrated with and part of its environment(s).

We have previously explored the use of $\Im$-open relative topology as a natural theory for neural nets [Heather and Rossiter 1991]. However topology is limited by its reliance on set theory and cannot really represent the integration that is essential for properties like emergence. Topology goes some way in this direction in products of compact spaces with the theorem of Tychonoff ([Kelley 1955] p.143) which is the subject of recent research for fuzzy topological spaces with intuitionistic logic [Çoker *et al* 2004]. This logic is important for open systems. The origin in the basic theory is that the complement of an open set is closed but the complement of a closed set may also be closed and not open [6]. A category theory explanation but still for sets is given by [Mac Lane and Moerdijk 1991] at p.50-57. Out of this gap comes intuitionistic logic escaping from the axiom of excluded middle.

The general power of topology and in particular the concept of openness was perhaps best appreciated by the Bourbaki group in France and their pursuit of non-locality led to the Grothendieck Universe as a universe of universes. However confined to set theory, this universe cannot quite attain to non-locality [7]. An open system's counterpart is *inter*connectivity but because

of the rather impoverished mathematical tools available for open systems. As just noted for there is no general theory on offer. Open systems are treated either informally or with piecemeal theory.

There is strong evidence that present technology is fast outstripping available theory, as for instance in the documentation of standards. The ISO family of standards intends an open system to mean intensional openness with its description of an open system as one that can be externally connected [8]. Nevertheless external and internal connections are linked enabling change at one place to affect happenings elsewhere. This extension-intension-extension non-locality needs a strong typing formalism to do it justice. Even more sophisticated thinking in system theory is exhibited at congresses of the International Federation for Systems Research where these ideas can only be expressed informally. Take for instance the understanding of an open system:

> Each and every model of system behaviour is a subjective interpretation carried out by an agent with cognitive capabilities in an environment ([Fredriksson and Gustavsson 2002] p.664).

Fredriksson and Gustavsson have no established theory to draw on in order to integrate formally these concepts of 'subjective interpretation' and 'agent with cognitive capability'. Alternatively those who want to use formal language have to take it example by example to define particular open systems. An instance of this is the open system definition given by Anatol Rapoport ([Rapoport 1986] p.176).

$$\frac{dx_i}{dt} = \sum_{j=1}^{n} a_{ij} x_i + c_i (i = 1, 2, \ldots, n)$$

This is a special case of a monomolecular chemical reaction modelled by a system of linear differential equations with constant coefficients, where x_i is the mass of substance i. Openness is derived from non-zero values of the c_i (representing the input or output of matter, energy or, in other contexts, of information). The system is closed if all $c_i = 0$.

This equation describes the change in mass of a particular mass x_i from the sum of some weighting of the mass measured from an origin c_i which may be dynamic arising from an additional loss of energy, mass or information. This seems the best that classical analysis can do to represent the effect of openness which itself is but the first step on the path of 'freeness' needed for a progression of naturality in the table of Figure 1, leading to inanimate global systems and animate living systems.

2 Category of Systems

The system theory for the 21st century needs to make formal these natural concepts of intraconnectivity, interconnectivity, intra-activity, interactivity, locality

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[5] As a section between ordered subsets the Dedekind cut and more elaborate version such as rough sets [Klir 2003] turn out to be but special cases of adjointness, the all-encompassing connectivity and activity described below.

[6] A fuller explanation from topology is at ([Kelley 1955] pp.44-45).

[7] Category theory suggests that an adequate model of

non-locality might be possible for the universe of universes of universes [Rossiter and Heather 2005].

[8] ISO/IEC 7498-1:1994(E) (p.3).

and non-locality. The theory needs to be realisable, that is, able to be constructed in the real world. This is where category theory is now available as a constructive mathematics providing a formal definition of naturalness for applicable categories. It is then possible to escape the constraint of locality in both time and space. Consequently reliance cannot be entrusted to axiomatic methods whether to the axioms of set theory or even to an axiomatic version of category theory after the manner that it is employed as a modelling tool in pure mathematics. Fortunately applied mathematics has the concept of process pioneered by Bergson and promoted by Alfred North Whitehead [Pearson and Mullarkey 2002]. Like much of applied mathematics, process category theory is technologically driven and generally awaits fuller exposition. The arrow of category theory [Manes and Arbib 1975] formalises for the first time the very eminent principle of constancy in change enunciated by Heraclites and Parmenides some two thousand five hundred years ago. The constancy is provided by the arrow with a common source and target.

2.1 Intraconnectivity

Such an arrow is normally referred to as where source and target are indistinguishable. Collections as entities can then be identified as such objects, and operations as arrows between objects. There may be many possible arrows between identity arrows as objects in Figure 2.

Figure 2: The Limit of Intraconnectivity between Identity Objects

It is a triumph of 20th century category theory to have shown that a unique limiting arrow may exist for all these possible arrows. It is known as the equaliser [9] and marked as the central arrow in Figure 2. These arrows represent the resulting intraconnectivity of a local system.

Figure 3(a) is a triangle where the apex as drawn represents the general entity or finite sequence of entities. Inaccessible entities are not part of the system by definition of the system. There is an ordering between entities given by the direction of arrows. This means that the arrow limit between two entities is also a limit of all possible paths between the entities. Each curvilinear arrow may be composite, that is intraconnect any number of (finite) identity objects as the curvilinear polygon of Figure 3(b). This polygon may be represented in an abstract form of the triangle with an apex representing general identity objects or a sequence of such. Identity arrows are omitted

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[9]Although not defined formally in this way by axiomatic categorists.

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from Figure 3 and each arrow depicted represents the limit of a family of arrows (as in Figure 2). Recall too that this is process category theory. The polygon or triangle is not Euclidean nor in fact embedded in any mathematical space whatever. The limit arrow is drawn as a straight line to indicate a geodesic according to the variational principles of mechanics.
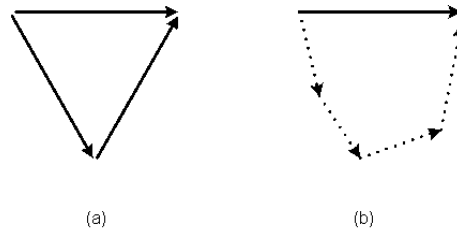

Figure 3: General Intraconnextivity Represented by a Triangle

But moving up a level there is a grand limiting arrow for all these limits existing as an identity functor characterising the type and therefore the system as a category. Because of the existence of limits and all possible connectivities, this is classified by axiomatic categorists as a cartesian closed category. This closure corresponds to the *local* condition in the last column of Figure 1.

A system as a category may then be drawn as in Figure 4 where the large circular arrow is the identity functor identifying the type of system. The triangle represents the curvilinear polygon just mentioned.
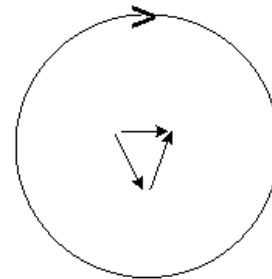

Figure 4: Identity Functor as the Intension of a Category-System

The system is therefore one large arrow i.e. process. All the internal arrows, triangles, polygons, etc are just components of this one arrow. The large arrow is the intension and the internal arrows are the system's extension. This then leads to interconnectivity between systems, that is a functor arrow between identity functors as the arrow marked F in Figure 6.

2.2 Interconnectivity

Where the two systems i.e. system-categories are to be distinguished there will be a functor arrow of some value other than an identity arrow. It may be noted that:

1. These two systems are of different types because they are distinguishable categories; nevertheless

the concept of system (like that of a category) is not distinguishable up to natural isomorphism.

2. This functor between the two categories is conceptually the same as the internal arrows between two identity arrows above

3. It is possible to repeat the abstraction to one higher level (a natural transformation) but no higher. This third level is that of interactivity to be dealt with below.

4. Because this is a self-closing type theory it avoids Russell's paradox. Furthermore as it is non-axiomatic we also avoid Gődel's Theorem of Undecidability.

However the functor between system-categories is more subtle than the simple description of the functor given above. For if we examine the fine structure of the arrow we find that in our cartesian closed category it can be resolved into the two functor arrows F and G, a relationship exhibiting the characteristic of adjointness [Lawvere 1969]. It is from this property that the openness of open systems arises leading to the further property of freeness and the formal notion of naturality [Rossiter and Heather 2002]. Figure 6 is itself a formal diagram of a more informal drawing by Rosen for system modelling (([Rosen 1991] Figure 7F.1) reproduced here in Figure 5. The category theory version is not a model and therefore avoids the modelling problem mentioned earlier.
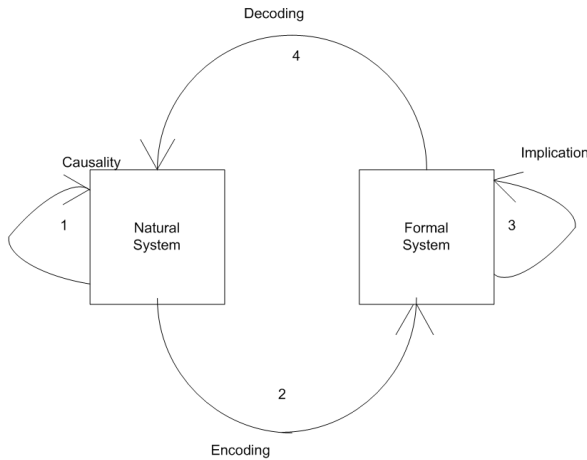

Figure 5: Adjointness from Rosen

2.3 Interactivity

Adjointness operates at every level but its typical significance is better shown at the level of Figure 6. The functor arrow from left to right can be resolved into two functors in opposing directions as in the diagram. The functor labelled F is the free functor and G is the underlying functor. Given any two of the left-hand category-system L, the right-hand category-system R, F and G, the other two out of the four are uniquely determined. This is an interconnectivity that gives rise to an interoperability written as the adjointness $F \dashv G$ where:
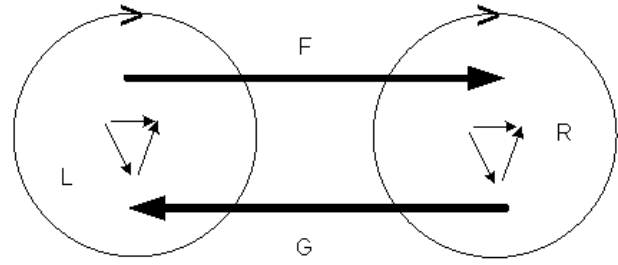

Figure 6: Interconnectivity between two Identity Functors leading to Interactivity between Category-Systems

$$\mathbf{1_L} \leq GF \text{ if and only if } FG \leq \mathbf{1_R}$$

Thus given the left-hand category-system L (expressed here in its intensional form of the identity functor $\mathbf{1_L}$) it is possible always to choose an appropriate F that generates any arbitrary right-hand category-system R (here $\mathbf{1_R}$) as desired. This is the significance and operation of the free functor F. The uniqueness arises from a naturality in the respective ordering of the two category-systems and their interoperability. This provides a formal definition of naturalness.

In the diagram of Figure 6 naturalness means there is only one solution for the respective triangles in the left- and right-hand category systems to match. Alternatively the right-hand category may be thought of as a definition of a free and open category-system where the freedom is provided by the choice of functor F and determined by the right-adjointness of the co-free functor G. This effectively does the job of the axiom of choice without the need for any assumption. Self-organisation of a category-system (intra-activity) arises when the category-system pair in Figure 6 are indistinguishable. The result could be viewed as the diagram in Figure 4 but with the enriched interpretation as an intension-extension relationship.

3 Conclusion

To sum up in a single word, a system is held together by *adjointness* – implicit for the pioneers of the local system in its environment but now made explicit in system-categories where Robert Rosen has led us. Adjointness defines naturality. Natural systems are free and open whereas in the 21st century cybernetic control needs to be in human hands whether at the macroscale of global environmental change and pandemics or at the microscale of biological nanoengineering and self-organising organs from stem cells in exact medicine. There is everywhere the need today to advance free and open systems theory.

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