Symposium 2: Applied Mathematics, Dynamical Systems, Logics and Category Theory

# Categorising Anticipatory Systems

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Categories, Naturality, Metaphysics, Intension, Data normalisation

Aristotle coined the word 'categories' to describe a structure of classes and it is used as the title of one of the books of his treatise on logic, the *Organon*. This begun a 2,000 year history for the category to describe the structured level at the foundation of logic. However when the study of logic diverted to the symbolic logic of set theory around 1900, the concept of a category with classes at various levels was no longer easy to represent. For the elements for the set are independent of one another and a set cannot be a member of itself so there was no inherent possibility to represent recursion nor relations other than by external functions. Nevertheless the category has become an indispensable component for many disciplines and the concept is still developing today. It is the primary classification system for Wikipedia which itself has about 500 types of categories defined. With advances in information systems the concept of typing is an aspect of categories that has increased in importance.

Thus there are two fundamental types of anticipatory system – the strong and the weak which differ markedly one from the other. The strong is a unique intension for any given configuration of the Universe. The strong anticipatory system is the real world one and needs therefore some metaphysical representation or at least a view at the level of the world itself. Strong anticipation is not then a property of an anticipatory system. Rather it is the essence of the system itself and requires impredicative mathematics. Weak anticipation on the other hand is a property of an anticipatory system and extensionally degenerate for any physical reality. It is predicative and may therefore be modelled in set theory. This mirrors the distinction in the category theory of mathematics which represents reality up to some isomorphism. The strong version is up to natural isomorphism and formalises the concept of 'natural' as found in the real world: the weak is only up to some assumed isomorphism. For instance the category of sets holds up to the isomorphism of Zermelo-Fraenkel set theory with the axiom of choice.

In category theory terms the anticipation of an anticipatory system resides in the relationships of its cartesian closed structure. That is the completeness of the whole which generalises the local completeness that Gödel proved for first order predicate logic of axiomatic systems. The cartesian closure provides the full formal rigour for strong anticipation. Weak anticipation models the strong. The subtypes of possible weak anticipatory systems can therefore be categorised by locally cartesian closed slice categories. These focus on material relationships for a particular context for the system. From the early days of information systems on computer it has been found convenient to characterise relationships in terms of dependencies on some key data. This makes possible a classification system based on degrees of normalisation in a database. This method of normalisation is difficult to justify in set theory but can now be given coherence by categorisation of the normalised dependencies. This paper examines in some detail the strengths and weaknesses of this method of categorising data normalisation as a prelude to applying it to classify weak anticipatory systems.