The Logic for Social Systems

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Abstract

Social systems are more complex than physical systems but systems theory and cybernetics are not extensible by adding local refinements as an incremental science. By general systems theory we would expect living systems to exhibit the same fundamentals as physical systems that can be expressed in logical terms, that is the language of mathematics, science and philosophy built on logic. As the complexity of systems increases so the theory has to dig deeper into these logical foundations to guarantee a rigorous application of its principles. This applies to the systems of human life to be found in biology and medicine, economics and social systems, that is if they are to have the power of the exact sciences. Human life resides in a myriad of connections at various levels. Possible interactions between levels lead to a more profound type of logic because the closed world assumption no longer holds at any level. It is necessary therefore to go back to first principles even to the work of Gödel (1906-1978) on incompleteness and undecidability for a natural logic. Social systems do not populate a Boolean world but a topos where the internal logic is Heyting.

1 Introduction

If ever there was a time for a theory to return to its roots - it is now for systems theory and cybernetics to make a rigorous examination of their logical foundations. The behaviour and control of interest are no longer simple in systems of the modern world. The study of how a system behaves from its parts is reasoning at a local level which is expressible in the first order predicate logic of classical mathematics which Gödel showed in his doctoral thesis [8] to be complete, that is internally consistent. This has proved very adequate for much science and engineering of the last century often by appealing to very clever models that approximated to first order predicate logic. This method has worked well in the past when the systems under examination were in general closed and the logic was that of a closed Boolean world. While the work of the early pioneers in systems theory and cybernetics like Ashby, von Bertalanffy and Wiener was well known and respected by the science community, nevertheless the mainstream was able to do quite well, thank you, without resorting to holistic concepts. Indeed there has been a lurking suspicion that the holism of systems theory has a touch of mysticism about it, which is not proper science and more like religion. Today applied science has shifted down into things like nanotechnology and across into intangibles like information science and how humans behave, none of which is any longer within the easy ambit of classical physics. Society and medical science are concerned not just with interoperating parts of a system but with the relationship between parts and the system as a whole and with interoperability between systems through increasing globalisation, including between parts of one system and parts of another system. The major difference is that these systems have to be treated as open [26] and therefore not conveniently accessible by first order predicate logic.

2 Open Holistic Systems

The consequence is that social science can no longer neglect today this side of systems theory and cybernetics. We all have to face up to the challenge even of that mystical quality and holism and pursue it in a scientific way. For phenomena like a living being or consciousness are themselves physical entities upon which classical methods have been able to make little progress. The mathematics and physics of life whether for the individual or groups of individuals have hardly been touched even within systems theory and certainly not in mainstream science. The energetic initiation in consciousness theory in the late 1990s came to almost nothing beyond that there was a hard problem. Systems theory suggests that new areas of the human and medical sciences will not get far without a full understanding, not only of entities like life and consciousness but even aspects of social and moral systems like 'conscience'.

Between intra-operability and inter-operability there has been a gap in causation that is between the parts and the whole. The mechanism can often be well described qualitatively but rarely logically beyond the rationalisation of a simple Boolean value. A substance injected into a cell may kill the organism as a whole but can provide no reason for the outcome. Scans of the brain may show parts affected but no enlightenment of what is going on. As artefacts become more complicated in their design and manufacture so they pass from simple objects to heterogeneous systems. The sequence of single instructions in the von Neumann computer is now replaced by the parallelism of an array of coprocessors with interoperating time domains [18] in modern commercial computers [19]. Human life resides in a myriad of connections at various levels, locally, nationally and globally. Possible interactions between levels in social structures lead to a more profound type of logic because the closed world assumption no longer holds at any level.

Furthermore systems of human behaviour have to be integrated from large high level social group activity down to the way individual genes and neurons operate. Examples of the application of quite sophisticated systems at the high level are such as credit risk between financial institutions, economics of money laundering, financial crime, global insurance industry, regulatory governments, international trade, money and wealth distribution, business activity cycles, trade competition policies, anti-trust law, full employment, urbanisation, entrepreneurial activity, health management systems, ageing, obesity, intellectual and industrial property rights, and computer security. At the low level examples can be found in the advances in bioengineering and neuroscience where there have been great advances recently in understanding the processes and mechanisms of innate behaviour such as sexual behaviour, olfaction, sleep and touch.

3 System Logic

It is with this growth in the sophistication of systems that we need to examine our foundations. Almost all those engaged in systems theory ascribe to the underlying truism inherited from the time of the enlightenment that to understand science is to understand it formally. Of the various possible forms of formalism, the best form must surely be the formalism of mathematics. It is somewhat of a paradox that if we dig deep enough into a foundation of mainstream scientists who have regarded holism as a touch mystical, that the tables are turned. It is the mainstream itself that rests on a foundation close to superstition. Eugene Wigner the Nobel laureate in physics has epitomised this effect by coining the phrase 'the unreasonable effectiveness of mathematics' [36] which was taken up and popularised by the computer scientist and coding specialist R W Hamming [15] so that it is a phrase oft quoted by many scientists with approval.

Wigner himself described 'the enormous usefulness of mathematics in the natural sciences as something bordering on the mysterious' and adds 'there is no rational explanation for it' ([36] at p 2). Hamming goes further to make it 'an act of faith on the part of scientists that the world can be explained in the simple terms that mathematics handles'([15] at p8). Wigner's stance seems to place current mathematical methods firmly alongside alchemy and astrology where the basis of authority is superstition. If our justification is no more than mathematics works then it is superstition no more and no less than those who still today trust their horoscope because it is justified by their experience. The justification of Hamming for mathematics is a little more sophisticated in that he makes it a matter of faith. That is he replaces superstition with religion. However this only begs the question in removing the issue one stage further away. This does not assist us because it would require us to be sure about the rationality of religion. In today's terms this would take us into further complicated questions like 'intelligent design'. Let us then keep within the formal approach and be guided by Gödel himself who was able to prove his incompleteness of mathematics using mathematics itself.

Wigner makes a very important second point: 'it is just this uncanny usefulness of mathematical concepts that raises the question of uniqueness of our physical theories'. However Wigner does not seek to answer this second question because he claims the data is not available. However it does give us a clue to assess mathematics beyond number and axioms. The subject matter of mathematics is continuously evolving usually driven by science and technology and Wigner's rather surprising attitude has even been extended to logic itself as applied to computer science [14].

This justification that current methods work may well be valid throughout the 19th and 20th centuries. But the question now is can we be sure that they will work as well for all the new sophisticated systems of the 21st century and beyond. To answer we need to have some appreciation of why they have worked in the past. A highly desirable feature required for free and open systems theory is exactness. As we shall see below exactness can be formally defined but may be informally interpreted as 'certainty'.

Probably the most rigorous path by which to approach certainty in logical foundations is through the work of Kurt Gödel that became a watershed in 20th century logic. There are two key concepts in Gödel's work which are components of 'certainty' and these are completeness and decidability. Gödel's 1929 doctoral thesis established that first-order predicate logic is complete [8]. This was followed the next year by his famous theorem of undecidability that applies to any system depending on axiom and number [1]. Gödel made three major contributions to logic that are very pertinent to the scientific method of the twentieth century. These are:

1. The system of first-order predicate (but not intuitionistic [11; 12; 13]) logic is complete [8; 9].
2. Any formal system of number and/or sets derived from axioms is undecidable [10].
3. Independence of the continuum hypothesis [3] [2].

[1] Gödel treated natural numbers and sets as equivalent because of the arithmetisation of sets [25]

[2] This is still an active research topic in proof theory

For such systems, cybernetic principles suggest a logic that permeates all three 'dimensions' of formal mathematics, empirical science and applied philosophy as enunciated by Husserl ([21] p.159; [1]) where just one or two on their own without all three together are woefully insufficient.

4 Logico-Mathematics

Husserl as a pioneer of post-modernism is still of interest today [2] but he wrote around the turn of the twentieth century at the time when the logistical approach to mathematics was in vogue. Mathematics and logic had just been merged by Frege and the fine detail was being hammered out rigorously by Whitehead (1861-1947) and Russell (1872-1970) in their *Principia Mathematica* [32] in the belief that logic underpinned mathematics and there was really no more to mathematics than logic. It was at that same time around the 1900s, as Husserl [21] was sowing the seeds of post-modernism, that David Hilbert (1862-1943) was advancing the cause of the formalist approach that mathematics was wholly regulated by the manipulation of formulae irrespective of their meaning or interpretation. To this end he was presenting a formal Programme (with 23 research problems) of mechanical logico-mathematics for the modern world. Difficulties were there from the outset like Russell's paradox to raise doubts on the sufficiency of both Frege's axioms and Hilbert's programme but it was left to Gödel in the early 1930s [9; 10; 11] by his two theorems of undecidability to disprove the hope that any mechanistic axiomatic system of logico-mathematical principles (as Gödel referred to them) based on number or sets could ever be found. Husserl was also proved right because there were two of his 'dimensions' missing - the science and the philosophy [3] [4].

We cannot apply Gödel's results properly without understanding logical foundations on which they are based. Gödel started with Russell and Whitehead's system [33].

The logico-mathematical basis for scientific reasoning is not clearly defined in mainstream work. If there is any consensus it is to be found within the tradition of Whitehead and Russell [33]. However, there is not even a standard version of these principles. For an analytical exposition of the principles of [33] it seems best to rely on the version given by Kurt Gödel. Because of the significance for all mathematical work and particularly because of applied mathematics for the rest of the twentieth century that rested on this foundation for reasoning itself, it is important to be aware of the nature of these principles consisting of formal axioms and rules of inference. Much if not all twentieth century mathematical models in science and engineering are postulated on them. They are nowhere uniquely defined but a typical list is given by Gödel himself as the starting point of his own work. He claims to rest on the propositions established by Whitehead and Russell denoted as *1 and *10 in their *Principia Mathematica*. Gödel reduced these to just eight axioms accompanied by four rules of inference ([8] p.67; [9] p.105). The four rules of inference are:

1. The inferential schema: from the truth of $p \wedge p \longrightarrow q$, there may be inferred q.
2. The rule of substitution for propositional and predicate variables [5].
3. The inference for universal quantification of predicates.
4. Individual free or bound variables may be replaced subject to scoping.

Whitehead and Russell themselves however point out that there are many implied assumptions along the way such as the meaning of truth and falsehood and indeed the *Principia* is subject to tentative qualifications throughout the original work and even more equivocation and variance is introduced in the later second [33] and abbreviated edition [35]. There are further alternative positions and qualifications put forward by Russell himself in his philosophical discussion of mathematics [27] and the book for the more general reader of *Introduction to Mathematical Philosophy* p.514 [29] [6].

A crucial principle in Whitehead and Russell's system of logic [33] is the Closed World Assumption with only the two Boolean possible outcomes. The upshot of these foundational axioms is that inference is defined only in terms of this Closed World Assumption. It means that negation, conjunction and disjunction are not independent. Although not mentioned by Gödel because he treats as given the assumptions of [33] nevertheless there are these fundamental definitions of true and false which are assumed by Whitehead and Russell. The first edition of the *Principia Mathematica* tells us we have to accept the concepts of truth, falsehood and the assumptions of the logical sum, logical product, complementarity and implication ([33] 1st ed. p.6). The later writings suggest that these four principles of deduction enumerated in [33] could be represented alternatively by five propositions ([29] p.149-150) although they do not explicitly correspond to those of Gödel.

relevant to scientific method for the concept of infinity and the meaning of an axiom [37].

[3] Gödel himself started to study Husserl in 1959 and subsequently recommends enthusiastically Husserl's investigation VI of the elements of phenomenological elucidation of knowledge in respect of categorial intuitions

[4] Gödel's self perception of his own social and intellectual role [30] in the situation in Central Europe can be gleaned from his replies to a sociology questionnaire in the 'Grandjean interview'. See Wang ([31]).

[5] Russell admits in footnote 1 p.151 [29] that no principle of substitution is enunciated in [33] 'But this would seem to be an admission'.

[6] There for example Russell picks up the point of CI Lewis in three papers in (Mind **xxi** 1912 p.522-531; **xxiii** 1914 p.240-247) on 'formal deducibility' as a narrower relation than strict implication. This is an important issue which leads to defeasibility [17], going further than the discussion of intuitionistic logic discussed here.

The second edition of [33] recognises that the four assumptions could be collapsed into one principle with the use of the Schaeffer stroke where $p \mid q$ is true if p is true or q is true or $p \wedge q$ is true, which is now further developed in the NAND operation. Whitehead & Russell [33] define as 'material implication' the concept $\neg p \vee q$ (See further Russell [28]).

The Closed World Assumption or to give it its older Latin tag *tertium non datur* (there's no third way) is relied on by the *Principia* and those who depend on its inference schema to define inference itself that is the assertion of implication $p \longrightarrow q$ from $\neg p \vee q$. Scientific models therefore that draw scientific inferences are assuming the Closed World Assumption with all its ramifications.

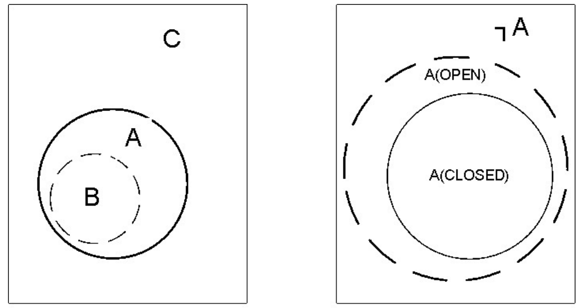
5 Logico-Science

As we have already seen to justify the use of scientific models because they work only holds where it is close to a first order model (which will then satisfy first order predicate logic) and problems arising from Gödel's theorems of undecidability can be avoided. The Scientific method of the last three centuries has actually achieved this by experimental verification. It is to be noted that this only holds locally and it is the completeness of first order predicate logic that gives such models their generality. For higher order and open systems experimental verification only holds locally without any guarantee of wider validity.

The paradigm of the scientific method is:
Theoretical model $\longrightarrow$ Prediction $\longrightarrow$ Experiment $\longrightarrow$ Validation
This model may well have been very adequate for the 19th century and it did lead to many ingenious and successful methods for bringing non-linear and higher order problems within first order models. This owes much to the influential work of Jevons who promoted a very general and weak concept of 'scientific induction' ([22] Lesson XXVIII, p.239-247). Aristotle has a full discussion on the subject of induction and as John Maynard Keynes points out (1883-1946) it is Aristotle's first meaning of the Greek word ἐπαγωγή as natural induction that is important, 'in which an abstract notion is exemplified' ([23] p.274).

In Figure 1(a) $B \subset A$ and if by inference we mean implication then A implies B. By the Closed World Assumption C is the complement of A. However in the real world system B is always part of its context C unless it can be fully partitioned off as a closed system. A full partitioning is not possible in practice nor allowed by the laws of physics but this situation can be represented by the open system. Figure 1(b) shows the difference between the closed (full circle) and the open (broken circle) situation. For a closed A then the complement of the complement of A gets back to closed A (i.e. classically $\neg\neg A = A$) but for an open A the complement of the complement of A gives closed A not open A (classically $\neg\neg A \neq A$). However classically $\neg\neg\neg A = \neg A$ even when A is open [7].

[7] This is an instance of the categorial principle that a three-level operation always provides a closure [18].



(a) (b)

Figure 1: (a) Classical Venn and (b) Open Venn Diagram

The open system leads to the intuitionistic logic of Brouwer. Gödel himself was clearly exercised in his mind by the work of Brouwer for he raised a qualification on only the second page of the typescript of his doctoral thesis ([8] p.60-61) referring to Brouwer before proving that first-order predicate logic is complete. Gödel however omitted the reference to Brouwer in the published version of his thesis. Feferman suggests that this may have been on the advice of his supervisor Hahn to omit Brouwer who was a *persona non grata* with Hilbert. Nevertheless Gödel went on to show in formal reasoning that first-order intuitionistic logic is not complete [11; 12; 13] but did show exceptional insight into this logic. Brouwer's student Heyting was able to show that it turns out very formally in the Heyting algebra [20]. It is this gap between open and closed systems that provides the non-classical third way needed for all higher studies including human affairs and socio-cybernetics. The real world is not a closed system which is the other side of the coin of constructivism and intuitionism.

6 Logico-Philosophy

We need therefore a basis for reasoning in the logic and we need a basis for understanding the science of socio-cybernetics in post-modern mathematics.

Husserl's own work led indirectly to the development of post-modernism in various artistic fields such as Derrida in literature culminating in a new general philosophic approach but not a formal one although Husserl himself started from geometry. The take-up for post-modernism in logic mathematics and science has in general been slow. The most promising is the process philosophy of Whitehead [34]. The logico-mathematical system alluded to above that began with Frege, was held out with great promise by Hilbert, rigorously presented by Whitehead & Russell but ultimately demolished by Gödel needs replacing by a philosophy that is free of the Gödel limitations. Rather curiously the current prime promise to meet the requirements was developed by Alfred North Whitehead. This is process philosophy [34]. It appears that while Whitehead and Russell were collaborating on the *Principia* they had their doubts about fundamental entities [18]. This leads to a formal philosophy, but a metaphysics not a model as distinguished by Pierre Duhem (1861-1916) [4]. A model suffers from Gödel uncertainty which is the common approach in theoretical computer science including artificial intelligence [8]. The Church-Turing thesis and recursive functions led to the von Neumann architecture which is still dependent on Gödel.

Category Theory provides a formal post-modern mathematics and brings together algebra, geometry and topology. It is fully formal in its logico-mathematical representation so far as it is based on the empirical scientific principles for the particular category known as cartesian closed and embodies this philosophy of process as understood by Whitehead. Category theory achieves and goes beyond the post-modern mathematics sought by the Bourbaki French School of Mathematics [9].

7 Category Theory

Category theory is based not on the set as a fundamental but on the concept of a morphism, generally thought of as an arrow and represented by $\longrightarrow$. The arrow represents the monoid of process whether a dynamic operation or a static condition and can cope therefore with descriptive/ prescriptive equivalent views. The cartesian closed category exists in a meta-physical sense with an instantiation of the structure of the real world both physical and social that we can know empirically.

Conventionally a category is a collection of arrows between objects which may be named corresponding to a type. An object is any variable entity identified by an identity arrow and some phenomenological datum. The ordinary arrows of a category operate between and relate distinguishable data. A category can then be considered as a type or class of datum with operations between data [10]. There is a recursive power to category theory not available in set theory because of the restrictive definition of set membership. A person could be a category and then its objects and arrows are characteristics of that person. On the other hand there is a category of persons when the objects are then individuals with the arrows providing the group dynamics.

A functor relates one category to another: it is a mapping between categories. In the example above where a person is a category then a functor would relate persons. Components of an information system may be represented by categories $\mathbf{A}, \mathbf{B}, \mathbf{C}, \ldots$ and with the functors between the categories recognised as knowledge. An arrow between functors is termed a natural transformation. If the objects are data, the categories information and the functors knowledge, then the natural transformations are wisdom, policy, etc.

Because a category is defined up to natural isomorphism there is a natural closure for cartesian closed categories at the level of a category of categories which exists by virtue of the recursive power of categories mentioned previously. A category of categories is known as a topos and its great significance for the context of sociocybernetics is that its internal logic is the intuitionistic as represented formally by Heyting. The real world as an interoperating open system is an instantiation of the topos and in particular gives us a formal description of how physical and social systems interact.

[8] For example in the use of agents [5; 6; 1].

[9] "Bourbaki's rejection of categories was one of the most significant points in the transformation of the group's spirit. For the first time, something that people knew to be eminently Bourbakian was mostly rejected out of a desire to advance without addressing .... the starting point."([24] p.84).

[10] An object in an object-oriented paradigm is not usually defined formally but seems to correspond to an object in category theory.

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