The Natural Metaphysics of Computing Anticipatory Systems

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Abstract

Anticipation is a natural characteristic of any system. 'Natural' is difficult to define formally in a mathematical model. For a model is an artificial construct relying on reduced conditions and assumptions. A model gives rise to weak anticipation while strong anticipation requires us to raise our sights to metaphysics where naturality resides. A prime distinction is that metaphysics has a higher-order tense logic. Strong anticipation is no prisoner of time like weak anticipation. In general while adjointness has a logical ordering the operation of an environment C on a subobject A has a solution subobject B under Heyting inference $A \Rightarrow B$ in the environment of C. This is represented as the expression $C \times A \longrightarrow B \dashv B^A \longleftarrow C$, the adjunction of the natural metaphysical ordering which constitutes strong anticipation. The environment C may be more particularised as an adjunction between the induced monad and comonad functors. The uniqueness of the adjunction in natural metaphysics is examined in the context of the Beck-Chevalley test for computing the multiplicity of formal models possible for weak anticipation.

1 Systems Theory

Anticipation is inherent in the natural relationship between category theory and systems. This is a hypothesis posed at the level of metaphysics.

Systems play an important role in many aspects of the information sciences. Yet despite some early attempts [27] the underlying theory has not been as well developed in formal terms as might be expected from the ubiquitousness of the concept. Furthermore many current application areas present science with challenges beyond a simple set-theoretic approach. Topical examples include pandemics, prediction of earthquakes, world finance, world energy management policy, regenerative medicine and climate change. Globalisation implies that local approaches are no longer sufficient and the need to relate one system to another suggests that freeness and openness are necessary properties in a systems approach ([18] at p.7).

The simplest view of a system, building on classical ideas, has been established by workers such as von Bertalanffy [7, 8] who define it as a collection of interconnected elements. Such a view regards a system as a closed entity with intra-connectivity between its component elements. This view is not ambitious enough for the current requirements of systems theory. In addition the term system has come to be closely related to another concept, the model. Checkland for instance defines a system as a model of a whole entity [9]; when applied to human activity, the model is characterised fundamentally in terms of hierarchical structure, emergent properties, communication, and control. The inclusion in this definition of emergence indicates the need for a data structure with the property of self-organisation, whose importance was earlier recognised as natural activity by Ashby [2]. Klir considers the system definition in terms of anticipation, which raises the question of interactivity between systems, that is a higher-order effect, requiring inter-connectivity. Such higher-order properties are an integral part of dynamic systems where the behaviour of one system is related to another [23]. The concept of a system involves a number of fundamental elements in the natural relationship between global and local, namely freeness, openness, connectivity, activity and self-organisation. These are summarised in Figure 1 from our earlier work [30].

system	**natural relationship**	**locality**
closed	*intra*-connectivity	local
open	*inter*-connectivity	local
self-organised	*intra*-activity	non-local
free	*inter*-activity	non-local

Fig. 1: Key Elements in the Definition of a System

2 Models

At first glance the term model has subtly different meanings from one area to another. For instance in mathematics a model gives meaning to sentences of a formal logic and in information sciences the model is an attempt to represent the structure and behaviour of the real-world through some notation. However a common theme is that the model attempts to add semantics to a structure, whether it be abstract such as a mathematical group or more concrete such as an information system. Many models are set theoretic with data structures defined as elements of varying complexity and the behaviour as transitions between states.

Much of the work to date on anticipatory systems concerns the comparison of present and future states but time is not essential to Robert Rosen's original notion of a system both predictive of and reactive to itself. We sought to show at CASYS'07 [16] that prediction is an attribute of predication. Not only is predication more general than prediction in time but it is more comprehensive of Rosen. His words

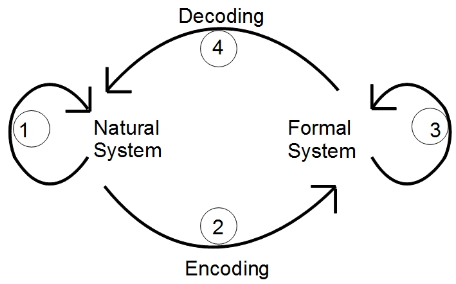

Fig. 2: Modelling a Natural System with a Free System, adapted from Rosen [28]

were 'A system containing a predictive model of itself and/or its environment'. He seemed to envisage three distinct aspects: the model of itself, the model of the environment and the model of itself and the environment. In Rosen's original diagram of Figure 2 we would interpret the model itself as ③ (implication), the model of the environment as ② ∘ ① (causality composed with encoding[1]) and the model of itself and the environment as ③ ∘ ② ∘ ① (causality composed with encoding composed with implication). The arrow ④ (decoding) is critical in composition for validation of the model as described later.

Dubois [10, 11] and others (including we ourselves) have investigated further to distinguish the two types of predication. The copula with the prediction and the reaction attributed to the system itself is interpreted as strong anticipation: the object as a proper model is interpreted as weak anticipation. Although not made explicit by Rosen this follows because a system cannot be a proper model[2] of itself. The 'model of itself' limb in his definition is therefore not modelling but is metaphysics. A model gives only partial predication whereas the metaphysical is full and complete. While a model loses information, metaphysics may add content to it.

This distinction in computing anticipatory systems has significance for a fundamental problem of philosophy in theoretical and practical aspects of computer science. The current Wikipedia entry for anticipation in the Rosen context concentrates on the issue of the need for an internal model in natural evolutionary cognitive systems. A more general and practical context for that issue might be on what representation of the real world should underpin information systems. Models in databases like the relational SQL or object-oriented operate as anticipatory systems for information retrieval.

These are applications where more attention needs to be paid to the role of the environment in Rosen's definition. A model of a system and its environment may not raise too many problems for weak anticipation but for strong anticipation the system

[1] *causality* and *encoding* are terms as used by Rosen but will be investigated further below.

[2] 'proper' follows from a model as a subset in set theory. For a set is always a subset of itself but not a 'proper subset'.

and its environment need to be integral. This may be only classic holistic systems theory but it is metaphysics not modelling. For a model cannot represent strong anticipation, only a model of strong anticipation. This suggests for applications in Artificial Intelligence the 'internal model' needs to be replaced by metaphysics. Iordache [19] uses the term metamodel to describe the metaphysical relation between category theory and real-world systems but of course 'metamodel' is only apt for the improper model where a system is a model of itself. A metamodel of a proper model is rather trivial. It is just the system itself. It seems preferable therefore to keep to the standard terminology of metaphysics/model as used by the French philosopher of science Pierre Duhem [13]. The use of the term 'metaphysics' may seem rather narrow as restrictive to physics but this is no bad thing as a reminder that in applied mathematics all metaphysical relationships are built-up on the relationship between physical objects of process.

3 Category of Systems

For a scientific understanding of systems and their engineering it is necessary to make formal all the connections and activity in Figure 1: intra-connectivity, inter-connectivity, intra-activity and inter-activity. The theory should be realisable, that is constructive, and should reflect the work on process categories by Whitehead [33]. Because of the continual interaction with a changing environment, the non-stationary has to be incorporated with the stationary. With the usual mathematical modelling tools, a set represents stationary objects. Non-stationary dynamics is provided by functions between sets but functions and sets are not integrated. To include natural living systems, where the interaction with the environment is literally vital, Rosen later proposed (as an early student of Sammy Eilenberg one of its founders) the use of category theory where both objects and mappings between them are interchangeable, each being representable by the same notion of the arrow [25]. Figure 2 is adapted from the original Rosen diagram of life ([28] Figure 7F.1).

Rosen's diagram can be simply interpreted as an exercise in the composition of his labelled arrows as ① = ④ ∘ ③ ∘ ②. That is the identity operation on the Natural System is equivalent to the composition of Encoding with the identity operation on the Formal System and with Decoding. Indeed Rosen observed that "When this is true, we say that the diagram commutes and that we have produced a model of our world". However, category theory enables us to elaborate further on the diagram's properties. For instance Figure 2 includes two identity functors[3] $①_{\mathfrak{NS}} : \mathfrak{NS} \longrightarrow \mathfrak{NS}$ and $③_{\mathfrak{FS}} : \mathfrak{FS} \longrightarrow \mathfrak{FS}$ for the Natural System and Formal System respectively. The identity functor, the intension for the system category representing the process [31], is shown as the arrow on the circle representing the category. The internal

[3]Gothic letters are used for the category names, indicating the categories are general, not restricted to small categories, that is the category of sets. Thus $\mathfrak{NS}$ is the general category of any natural system and similarly for $\mathfrak{FS}$, any formal system.

arrows of the category are then the extension as a curvilinear polygon as shown in Figure 3. The diagram shows a system with intraconnectivity.

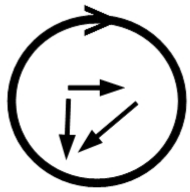

Fig. 3: Identity Functor as the Intension of a Category-System

Figure 2 also exhibits interconnectivity. The arrow ② (Encoding) is a functor from $\mathfrak{NS}$ to $\mathfrak{FS}$ written ②: $\mathfrak{NS} \longrightarrow \mathfrak{FS}$ and the arrow ④ (Decoding) is a functor from $\mathfrak{FS}$ to $\mathfrak{NS}$ written ④: $\mathfrak{FS} \longrightarrow \mathfrak{NS}$. We now have two systems $\mathfrak{NS}$ and $\mathfrak{FS}$ with the interconnectivity relationship between them represented by the functor arrows ② and ④ as in Figure 4.

Fig. 4: Two-way Mapping of Functors ② and ④ between categories $\mathfrak{NS}$ and $\mathfrak{FS}$

If the formal system is a perfect representation of the natural system then we can map forward with ② and backwards with ④ without loss or gain of information. In this case the natural system and formal system are isomorphic. However, in practice it is much more likely that there will be imperfections in the representation of $\mathfrak{NS}$ by $\mathfrak{FS}$. If certain conditions are satisfied we can still though have a relationship between the two functors in which the displacement of the mapping into each category is measured. This relationship, termed adjointness, was first recognised by Kan [21] and its understanding in category theory was advanced by Lawvere [24] to show that syntax and semantics are a pair of contravariant functors. We now know from advances in information systems that this can be further generalised into intension and extension but mutually with respect to each other to be dealt with below.

Figure 5 therefore shows two systems with interconnectivity between them. The diagrams can be re-drawn to show how an object[4] in the left-hand category FS (that

[4]therefore presented as in Roman not Gothic font.

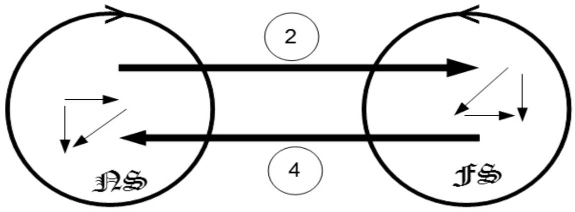

Fig. 5: Interconnectivity between Categories $\mathfrak{NS}$ and $\mathfrak{FS}$

is $1_{FS} : FS \mapsto FS$) is related to ④ $\circ$ ②(FS), that is the result from mapping an object in $\mathfrak{FS}$ to the right with ② and then back to the left with ④. The relationship between 1_{FS} and ④ $\circ$ ②(FS), if adjointness holds, is given by η, the unit of adjunction. If η is the initial object $\perp$ then no change has occurred in the mapping and the relation is the special case of isomorphism. Similar reasoning can be applied to the right-hand category $\mathfrak{NS}$ on the relationship, ϵ the counit of adjunction, between $1_{NS} : NS \mapsto NS$ and ② $\circ$ ④(NS). These more detailed mappings are shown in the diagrams in Figure 6 where Figure 6(a) shows the displacement in the left-hand category $\mathfrak{NS}$ when the unit of adjunction η is other than $\perp$, the initial object, and Figure 6(b) shows the displacement in the right-hand category $\mathfrak{FS}$ when the counit of adjunction ϵ is typed other than by T, the terminal object. The two functors are adjoint if the two triangles in Figure 7 commute, that is in (a) ④$(g) \circ \eta = f$ and in (b) $\epsilon\circ$ ②$(f) = g$. If the two triangles do not commute, then the functors ② and ④ are not adjoint and cannot therefore exist naturally.

4 The Intension/Extension Relationship

Relationships in nature are explicable in process categories with the single concept of adjointness [24] that consists only of a pair of contravariant arrows inducing a monad. In finitary categories the mathematics of adjointness has been developed in what is termed a cartesian closed category, derived as an abstraction of the cartesian product but this description from historic origins may by its simplicity mislead as to its great power and content. The finitary approach is to distinguish the two properties of cartesian closed and locally cartesian closed but in process categories it is that natural distinction between intension and extension.

In cartesian closed categories everything in the world is related to everything else in the world. The formal structure of the relationship may therefore be relevant to any scientific study or technological application requiring an understanding of these relationships. The intension-extension relationship is fundamental as it provides for the intension at one level to define the permitted instances at the next. The intension-extension distinction is implicit in Aristotle's *Organon* but not really made

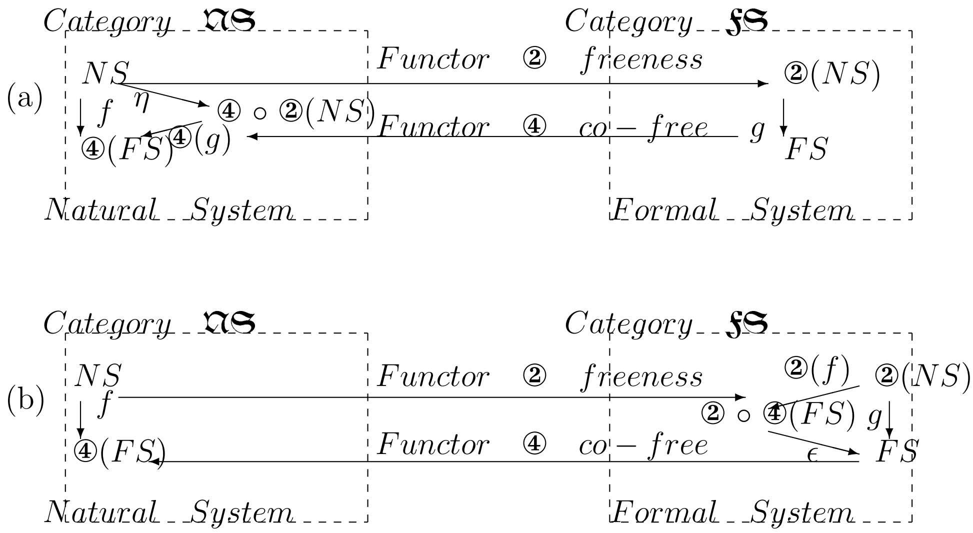

Fig. 6: Adjointness between Natural and Formal Systems: (a) unit of adjunction η other than $\perp$, (b) counit of adjunction ϵ other than T

explicit until brought out in the Port-Royal logic of 1662-1683 [1].

Process categories as a metaphysics provide mixed levels for intension/extensions. Intension and extension alternate in a pre-order, that is with an arbitrary beginning [29]. This is the natural role of the arrow in category theory with an identity arrow as intension and a distinguishable valued arrow for extension. The simplest identity arrow is treated as an object, the next higher identity arrow (the functor) composed of extensional arrows between objects makes a category with ordinary functors as extensional arrows between categories. The highest level arrow is the natural transformation which composes structures of categories and functors with the identity natural transformation constituting a topos. The whole is therefore a recursive system with closure at four levels [29] consisting of three open interfaces as shown in Figure 8. This is process and the Universe is an instantiation of process but the World is even greater than the physical Universe for the World consists of all the relations between physical entities and all the relations between those relations.

The diagrams so far show in an abstract way a two-way mapping between one intension-extension pair $\mathfrak{NS}$ and another $\mathfrak{FS}$, each represented as an identity functor (in terms of Rosen's analysis). The question is whether the mapping can be shown to reveal a greater level of detail.

Figure 9 shows for a cartesian closed category the relationship between intension and extension in terms of the functors Σ (exists), Δ (diagonal or pullback) and Π (product). Σ is left adjoint to Δ and Π is right adjoint to Δ. Δ plays a dual role in that it is both right adjoint to Σ and left adjoint to Π. The relationship is often written $\Sigma \dashv \Delta \dashv \Pi$, indicating the two adjunctions involved. Figure 9 shows

$$
\begin{tikzcd}
NS \arrow[r, "\eta"] \arrow[d, "f"'] & \textcircled{4}\circ\textcircled{2}(NS) \arrow[dl, "\textcircled{4}(g)"] \\
\textcircled{4}(FS) &
\end{tikzcd}
\qquad \text{(a)} \qquad \text{(b)} \qquad
\begin{tikzcd}
& \textcircled{2}(NS) \arrow[dl, "\textcircled{2}(f)"'] \arrow[d, "g"] \\
\textcircled{2}\circ\textcircled{4}\ (FS) \arrow[r, "\epsilon"] & FS
\end{tikzcd}
$$

Fig. 7: Roles in Adjointness of a) η, the unit and b) ϵ, the counit of adjointness respectively

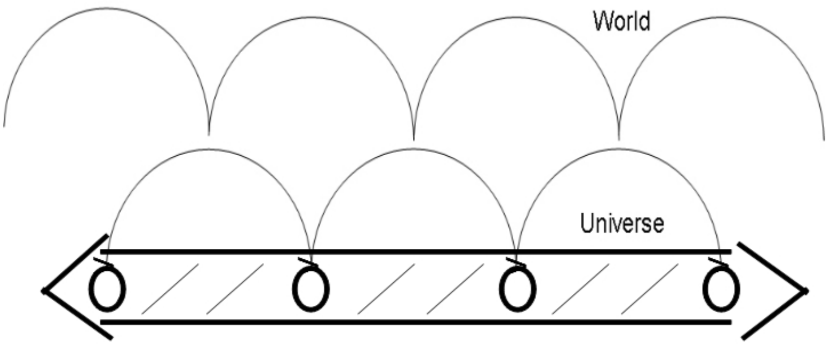

Fig. 8: Four-level Description of the World and the Universe

the alternation between intension and extension with the input to Σ intension, the output of Σ and the input to Δ extension, the output of Δ and the input to Π intension and the output of Π extension. Intension is left exact to extension and extension is right exact to intension[5].

Figure 10 shows for a cartesian closed category an explosion of the intension-extension relationship $\Sigma \dashv \Delta \dashv \Pi$. Σ, Δ and Π in the categorial diagram of Figure 9 apply to each source object in the domain. Functor Σ identifies each product in the category and assigns them to B. Functor Δ picks out the arrows from A to product B. Functor Π identifies all arrows from $A \longrightarrow B$ to C. So composition of $\Pi \circ \Delta \circ \Sigma$ to give F only holds if the following all hold also:

1. There exists a product $C \times A \longrightarrow B$.

2. There exists an exponential $B^A \longleftarrow C$.

[5] Adjointness and exactness for right and left must be carefully distinguished [17].

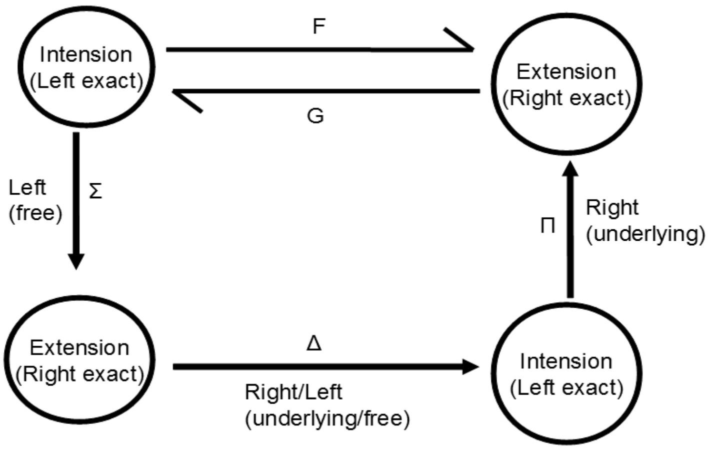

Fig. 9: Relationship between Intension and Extension in terms of the Functors Σ, Δ and Π

3. There exists a universal $C \longrightarrow C$.

For a category to be cartesian closed it satisfies these conditions for F and moreover the adjointness $F \dashv G$ holds such that $\Sigma \dashv \Delta \dashv \Pi$.

In general, adjointness gives a logical ordering: *iff* the operation of an environment C on a subobject A has a solution subobject B then the Heyting inference $A \Rightarrow B$ applies in the environment of C. This can be represented as the adjunction:

$$C \times A \longrightarrow B \dashv B^A \longleftarrow C$$

This adjunction is the natural metaphysical ordering which constitutes anticipation. Thus causation (left adjoint) and Heyting inference (right adjoint) are both stationary forms of the predicate[6] of anticipatory systems. Moreover these adjoints dominate the two mainstream applications of AI and databases. In AI the left adjoint is a relevance connection in context and the corresponding right adjoint is cognition. For information science including data warehousing, data mining and the semantic web, a query in context is left adjoint and the resultant retrieval right adjoint

In the next section we explore the conditions for anticipation in more detail, in particular by applying Rosen's example in an expanded environment, using Beck's monadicity theorem.

[6]'predicate' is used in the same sense as in 'predicate calculus'.

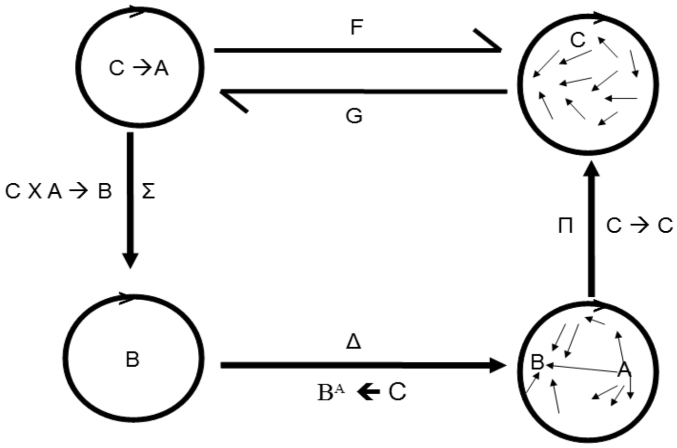

Fig. 10: Cartesian closed category: Explosion of Intension-Extension Relationship $\Sigma \dashv \Delta \dashv \Pi$. For simplicity of terminology subscripts in natural transformations, indicating components ([25] at p.16), are not shown

5 Intension-Extension Relationship in Terms of Rosen's Model

Rosen's example in Figure 2 can be reinterpreted in terms of cartesian closed categories as in Figure 11. The free functor ② takes the category for the formal system $\mathfrak{FS}$, representing the intension, to the category for the natural system $\mathfrak{NS}$, representing the extension. The underlying functor ④ takes the category $\mathfrak{NS}$, the extension, to the category $\mathfrak{FS}$, the intension. If adjointness holds we can write ②⊣④, indicating that ② is left adjoint to ④ and ④ is right adjoint to ②.

From the explosion perspective there is an alternation of intension and extension. The terms are relative rather than absolute with an intension-extension pair at one level mapping into another intension-extension pair at the next level down. For a cartesian closed category the adjointness ②⊣④ can be decomposed into $\Sigma \dashv \Delta \dashv \Pi$ where the functors are respectively $\Sigma : \mathfrak{FS} \longrightarrow \mathfrak{NS}$, $\Delta : \mathfrak{NS} \longrightarrow \mathfrak{FS}$, $\Pi : \mathfrak{FS} \longrightarrow \mathfrak{NS}$. Note the subtlety of Δ: Δ is right adjoint to Σ and left adjoint to Π. So, while Σ is a free functor and Π an underlying functor, Δ can be viewed as both a free and an underlying functor.

The identity functors ① and ③ for the categories $\mathfrak{FS}$ and $\mathfrak{NS}$ respectively are of particular interest. A cartesian closed category has an identity functor. So this requirement means that the individual categories are cartesian closed as well as participating in the adjoint intension-extension relationship. We can then write for the category $\mathfrak{FS}$ the identity functor $1_{\mathfrak{FS}} : \mathfrak{FS} \longrightarrow \mathfrak{FS}$ and for the category $\mathfrak{NS}$ the identity functor $1_{\mathfrak{NS}} : \mathfrak{NS} \longrightarrow \mathfrak{NS}$, ① and ③ respectively in Rosen's labelling, as in Figure 11.

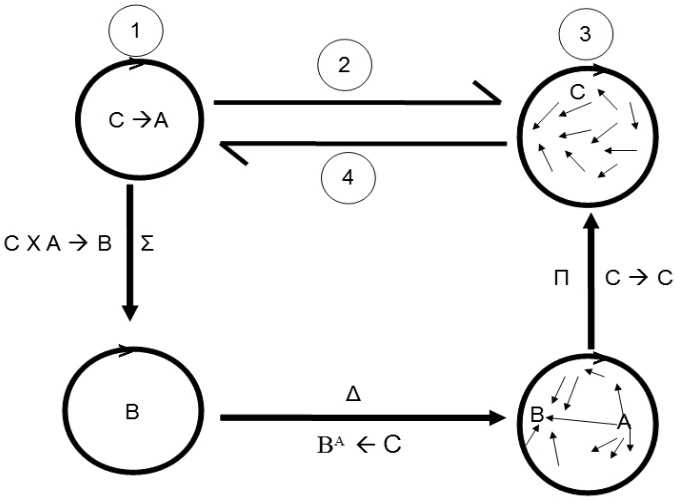

Fig. 11: Intension-Extension Relationship for Natural Metaphysics as Cartesian closed categories

Figure 11 represents natural metaphysics with the formal system the intension and the natural system the extension. Adjointness gives the unique state when all the mappings satisfy the conditions in Figure 6. Anticipation compares one state with another in both a forward and backward direction. It is the generalisation of a differential in classical Calculus. The monad construction and its dual the comonad provide the categorial generalisation of anticipation [31]. The associative laws for a monad are shown in Figure 12(a). Taking the adjoint pair of functors ②∘④ as the endofunctor T we can compose the diagram in Figure 12(a) to compare the states $\mu : T^3 \longrightarrow T^2$ and $\mu : T^2 \longrightarrow T$. The arrow μ, the multiplier, compares T between one state and another by looking back at the previous value. For the comonad, whose associative properties are shown in Figure 12(b), the endofunctor is S (④ ∘②) and the states compared are $\delta : S \longrightarrow S^2$ and $\delta : S^2 \longrightarrow S^3$. The arrow δ, the comultiplier, compares S between one state and another by looking forward to the next value. The categories for monads and comonads are historically described and written as triples[7] ([6] section 3 pp.83-122, including pp.121-122 for historical notes on the usage of the term): $\mathbf{T} =< T, \mu, \eta >$ and $\mathbf{S} =< S, \delta, \epsilon >$, respectively, where η and ϵ are the unit and counit of adjunction, respectively, as already introduced.

Anticipation might be thought of as being measured by δ, the change looking forward. But this is too simplistic. Forward and back refer to the natural ordering of which time is a special case where 'anticipation' is often considered. But it is a much deeper and more comprehensive concept in general. There is a further possible

[7]Set theoretic terms such as triple, unit and multiplier are often used for monads which are not really general enough for monads induced by general categories. We use these words because we have no others.

$$
\begin{tikzcd}
T^3 \arrow[r, "T\mu"] \arrow[d, "\mu T"'] & T^2 \arrow[d, "\mu"] \\
T^2 \arrow[r, "\mu"'] & T
\end{tikzcd}
\qquad
\begin{tikzcd}
S \arrow[r, "\delta"] \arrow[d, "\delta"'] & S^2 \arrow[d, "S\delta"] \\
S^2 \arrow[r, "\delta S"'] & S^3
\end{tikzcd}
$$

(a) (b)

Fig. 12: Associative Law for (a) Monad $< T, \mu, \eta >$, that is $\mu \circ T\mu = \mu \circ \mu T$; (b) Comonad $< S, \delta, \epsilon >$, that is $S\delta \circ \delta = \delta S \circ \delta$

adjunction between the functor parts of the monad **T** and the comonad **S**, that is $T \dashv S$ ([6] at p.120). The conditions for this adjunction to hold have already been given at a basic level in Figure 7 but can be re-stated with more relevance for the monad/comonad relationship in the series of diagrams developed below.

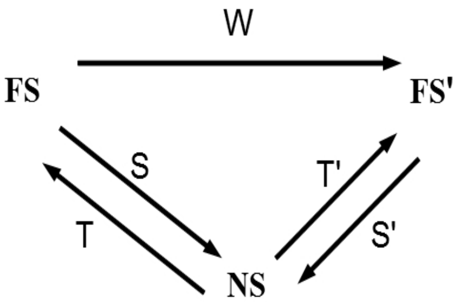

Fig. 13: Relationship between one Formal State FS and another FS$'$ in the Context of a Natural State NS and two potential adjunctions $T \dashv S$, $T' \dashv S'$ where FS and NS are states being subobjects in their respective categories of $\mathfrak{FS}$ and $\mathfrak{NS}$

In Figure 13 the looking forward from the natural system $\mathfrak{NS}$ to the formal system $\mathfrak{FS}$ is performed by T the monad functor with S the comonad functor looking back ([6] at p.115). The monad functor T has source of NS and target of NS; the comonad functor S has source of FS and target of FS. The functors may be described as endofunctors as the source and target are isomorphic. However, the monad and comonad both involve an intermediate category: T maps back to NS via FS; S maps back to FS via NS. We therefore have the arrows:

T : NS $\longrightarrow$ FS $\longrightarrow$ NS as the composite of F : NS $\longrightarrow$ FS with G : FS $\longrightarrow$ NS

$S : \mathrm{FS} \longrightarrow \mathrm{NS} \longrightarrow \mathrm{FS}$ as the composite of $G : \mathrm{FS} \longrightarrow \mathrm{NS}$ with $F : \mathrm{NS} \longrightarrow \mathrm{FS}$

As the formal system moves from one state to another $W : \mathrm{FS} \longrightarrow \mathrm{FS}'$, the mappings between the formal system and the natural system change from S and T to S' and T' respectively, as also shown in Figure 13. If the diagram is to be a category it must commute so that $T' \circ S = W$. The change in the formal system $W : \mathrm{FS} \longrightarrow \mathrm{FS}'$ is the required anticipation. However W is rather open at this stage with no preservation of limits or colimits.

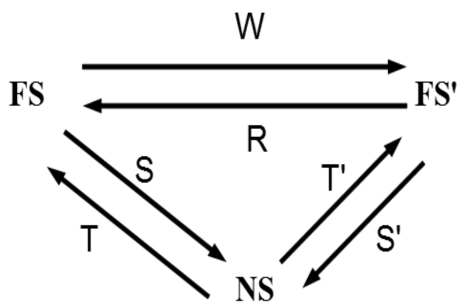

Fig. 14: Anticipation between one Formal State and another as the adjunction $W \dashv R$ in the Context of Figure 13

Closure can be provided by introducing a further arrow $R : \mathrm{FS}' \longrightarrow \mathrm{FS}$, dual to W as shown in Figure 14. For the diagram to commute there are now two equations: $T' \circ S = W$ and $R = T \circ S'$. If W is left adjoint to R and R is right adjoint to W we write $W \dashv R$, with W preserving colimits and R preserving limits. Anticipation is given by W with the conditions that the diagram commutes, that the monad/comonad adjunctions $T \dashv S$, $T' \dashv S'$ hold and that $W \dashv R$. Such anticipation is potentially strong as the environment and the system are tightly integrated.

Diagrams such as that in Figure 14 are familiar in category theory as Beck's monadicity[8] theorem ([6] p. 101, 115). Beck developed his ideas in the 1960s with Barr [3, 4], showing the conditions for adjointness between the induced monad and comonad functors. Mac Lane ([25] pp.149-155) provides a more current interpretation of the coequalizers in terms of colimits. From the point of view of the present work such diagrams can be considered as slice categories, in which the relationship between FS and FS$'$ is considered in the context of a third subobject NS. Such categories are locally cartesian closed, hence expressing the intension-extension relationship as in Figure 11.

A categorial integration of the two approaches is presented in Figure 15 where the relationship between one slice category and another is given by the two pairs

[8]formerly 'precise tripleability'.

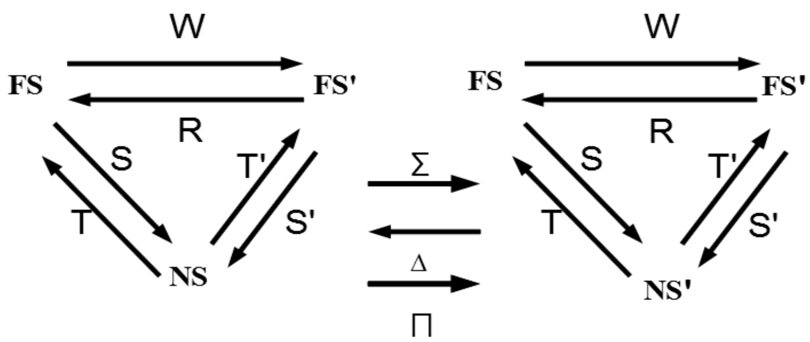

Fig. 15: Intension-Extension Relationship between one Slice Category and another as the Adjunction $\Sigma \dashv \Delta \dashv \Pi$

of adjoints representing an intension-extension relationship $\Sigma \dashv \Delta \dashv \Pi$. In the left-hand slice category the mapping is onto the initial state of the natural system NS; in the right-hand slice category the mapping is onto the next state of the natural system NS$'$. W, mapping the formal system from one state to another, strongly anticipates the change in the natural system between the intension, the left-hand slice category, and the extension, the right-hand slice category.

6 Intension-Extension Relationship in Natural Metaphysics

Rosen never pursued his category theory suggestion before his untimely death. Because of the strict rigour of category theory an implementation shows that Rosen is operating simultaneously at more than one level in his diagram of life in Figure 2. Category theory suggests that the essence of life resides in the natural metaphysics. His diagram should therefore be expanded recursively as in Figure 16. The contrast between the natural computing of metaphysics and formal models of computing is apparent in the different manifestations of mathematics as a formal language. To be natural the metaphysics of process philosophy restricts categories to those that are cartesian closed. This provides the unique adjointness of $T \dashv S$ as the intensional limit with an existence of possible extensional colimits by means of the free functor. Each extension is still unique according to the value of its respective adjointness $< \eta, \epsilon >$. Finitary category theory on the other hand admits arbitrary categories modelled in the category of sets. This is the usual version of category theory that is used for formal purposes although the subject itself evolved from algebra, geometry and topology as a higher level mathematical workspace. However the category of sets requires axioms and is therefore undecidable.

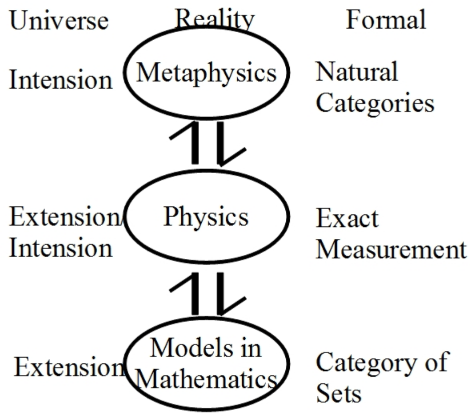

Fig. 16: World Intension/Extension in Physics and Mathematics

That gives scope for variety but is therefore degenerate, for example sketches [20, 5]. Open systems represented by arbitrary categories are not necessarily complete. Where it is possible to induce a triple, adjointness is present but gives rise to the need to test for 'tripleability'. The early unpublished work in the mid-sixties mentioned above by Jon Beck working with Michael Barr [6, 25] established the sufficient conditions for tripleability. Beck's conditions to induce adjointness (i.e. what he called 'tripeleability') require the existence of a stability functor that is both free and underlying. It is a functor that[9]:

1. has a left adjoint;
2. reflects isomorphisms; and
3. preserves coequalizers

The first test establishes intension, the third establishes extension and the second provides that their respective entities are naturally isomorphic (ie 'the same' for practical purposes). Around the time of Beck's work the existence of the higher order cartesian closed category of the topos was emerging. This provides for an adjointness between the triple and its co-triple and is found to have an open internal logic where the Boolean world is replaced by its intuitionistic counterpart from Heyting algebra. The 'tripleability' was updated to 'monadicity' but substance also needed to be enhanced by incorporation of the openness of the Heyting logic. This

[9]Because of the limitations of the category of sets a number of technical points need to be added to the sufficient conditions leading to greater complexity ([6] at pp.117-121) than in the full process category version

is again[10] an example of the need for greater complexity for arbitrary categories (see for instance [26], [20] pp. 589 *et seqq*). For it was soon apparent that the Boolean world had a relevant counterpart in the Zariski open topology developed by Grothendieck. In particular the work of Claude Chevalley in the Bourbaki group of French mathematicians on constructible sets in algebraic geometry, is able to complete the internal logic for Beck's theorem to provide what has become known as the Beck-Chevalley conditions for stability. In terms of Figure 10 these criteria relate to the dual nature of the pullback functor Δ. For cartesian closed categories in process theory the significance of the Beck-Chevalley conditions are no more than the straight forward test for adjointness. However it does underline the subtle duality of the stability of Δ which is quite profound in the way that it switches the free and underlying functors in the intension/extension relationship. It is also the critical point of the test for a category to be locally cartesian closed and explains and provides the relationship between different local extensions for the same intension that is the global/local relationship. An example is the stability between slice categories [15].

7 Conclusion

To sum up anticipation is the property of any natural system $< \mathbf{T}, \mathbf{S} >$, computed from its monadicity $< T, \mu, \eta >$ and comonadicity $< S, \delta, \epsilon >$ and given by $W \dashv R$ (in diagram 15) where δ is the measure of the prediction typing forward ($S \longrightarrow S^2$) and μ the predication back typing ($T^2 \longrightarrow T$). It should be noted that the detail of this analysis refers to strong anticipation.

The detail for the lower extension in Figure 16 is mainly concerned with weak anticipation which has special considerations. We hope to present at a future CASYS a study on classifying information systems by weak anticipation.

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