Formal Order and the ‘Loveliness’ of Nature

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Talk to 6th European Summer School in Process Thought
Natural and Artistic Beauty in Process Metaphysics
Katowice, Poland, by Zoom, 6 August 2021

Acknowledgements

Musicians who have inspired my interest in the art and helped me to understand it

Whitehead and Nature

- Alfred North Whitehead (1882-1947) devoted a long life to study the nature of Nature which he called ‘loveliness and power’ (Adventures of Ideas (AI),1933, p19).
- In his later period he recognised ‘the welding of beauty to regularity of geometrical form’ (AI 1933, p124) but realised that this formed part of a radically different cosmology of events that he could best represent by abandoning his former descriptions in mathematical terms.

The Nature and Beauty of Music

- Cultural Environment
  - Part of all cultures
  - Expressiveness
  - Skills
- Complex physical sounds
  - Handling all emotions
  - High degree of variation in genre
  - Audience participation

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Combine the Beauty of Music and Maths

- Music
  - Formal representation and order (in some cultures)
  - Expression
- Mathematics
  - Category Theory
    - Formal relationships, including order
    - Complex multi-level mappings (expressiveness)

Metaphysical Language – Adjunction

- Category Theory is itself relevant as a metaphysical language that has brought to the fore the existence of universal limits and co-limits that are formally connected by adjointness.
    - That is the generalised relationship between syntax and semantics for contravariant endofunctors F, a free functor and G, the underlying functor with the adjunction written as F-| G

Metaphysical Language – The Classical Topos

- The topos is based on the idea of Aristotle for tackling a legal argument with the premises held within its structure and the logic returning true or false as the outcome.
- In category theory the classical topos as defined by Grothendiek and others is closed at both ends and the truth object or subobject classifier may be more complex, for instance based on the natural numbers.

Times and Plus

- At the lower end there exist products of objects, connected by times X, and a limit.
- At the upper end there exist coproducts of objects, connected by sum +, and a colimit.
- Limit is greatest lower bound
- Colimit is least upper bound
- Interplay between X and + plays a critical role in categorial applications
- This is the Cartesian World

Real-world Topos

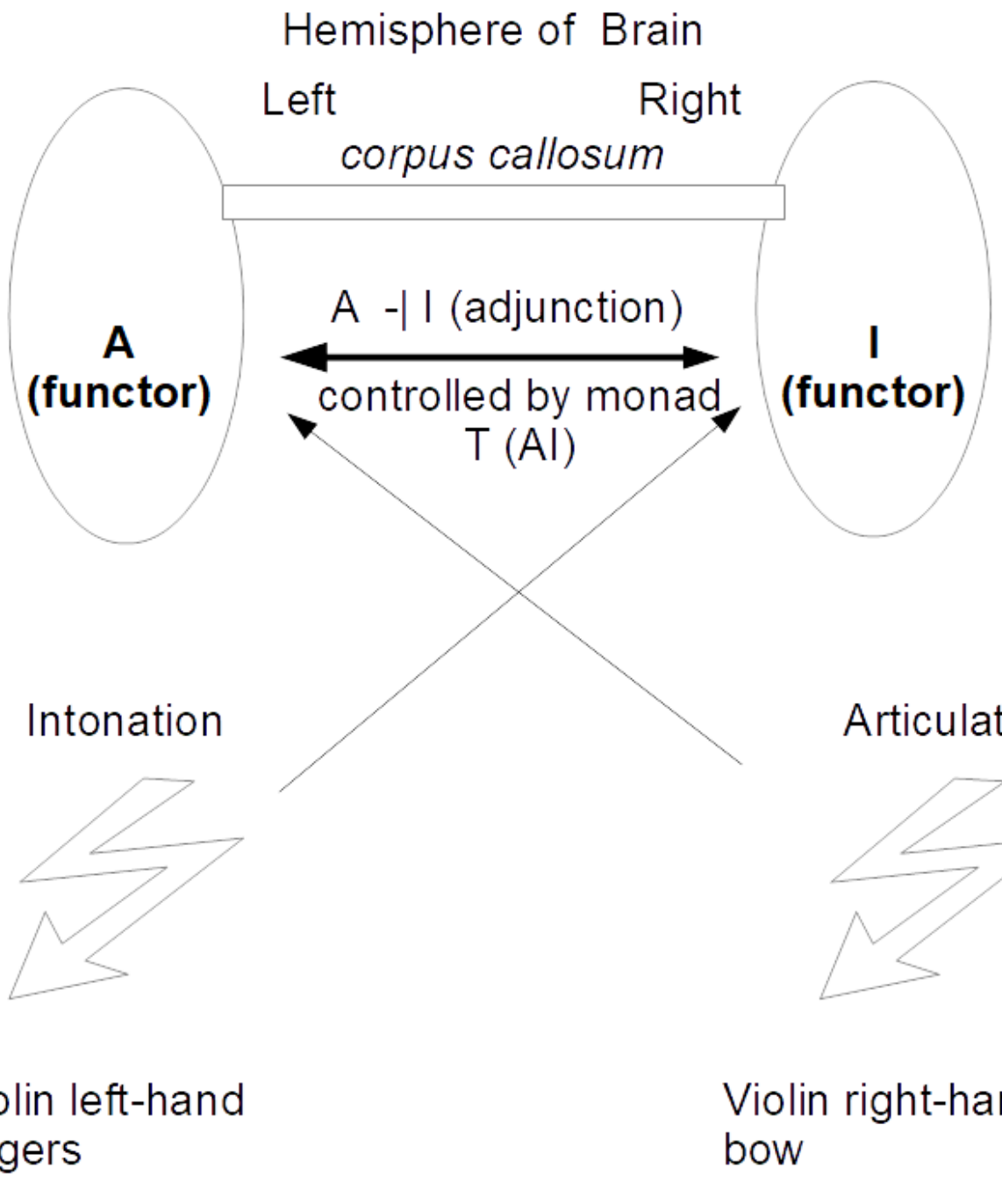
- If either the limit or colimit does not exist, then the category is not a classical topos.
- The existence of the colimit requires a single (unique) arrow from it to every object in the topos
- If the arrow is not unique then the colimit is said to be weak and the structure is not a topos
- We are exploring this condition in our work
- The colimit is the initial object of the topos

Example of Music 1

- Recent work by the authors has concentrated on the example of music in category theory
  - logical aspects such as
    - players and scores
      - sections of orchestra, soloists, conductors
      - composers, score variants
    - occasions, representing a co-ordinated sound by the performers at a particular timeline
    - even administration of the concerts
- Performance has been of particular interest
  - Monad as process represents movement from one timeline to another

Example of Music 2b

- Monad (after Leibniz) AI emulates musical processes in the brain for
  - Adjunction AI or <A, I, η, ε>
    - A (Articulation) is free functor (lhs of brain)
    - I (Intonation) is underlying functor (rhs of brain)
    - η, ε are unit/counit of adjunction for offsets in mapping
  - Example is clearest for violin with
    - Bowing by right-hand feeding into lhs of brain
    - Finger control by left-hand feeding into rhs of brain
- Monad takes one (adjoint) step through the timeline. Monads may be composed naturally.
- Also dual Comonad IA



Monad -
3 cycles

Process-thought

Published

- Rossiter, Nick, & Heather, Michael, Musical Performance: a Composition of Monads, Sociology and Anthropology 7(4), 178-188 (2019)
- Rossiter, Nick, & Heather, Michael, Physical Sounds as Colimits in the Topos under Monad Control, ANPA 40, Aporia, 40th Anniversary Proceedings 1979-2019, John Ceres Amson (ed), 541-578 (2020)

Physical Sounds as Categories

- This paper will take the work forward by exploring how
  - physical sounds are defined in terms of a topos
  - the conditions occur for colimits to exist
  - a simplification of the formalism presented in the 2020 ANPA paper could be made in certain circumstances

In Mind

- Always interested, as computer scientist, in computer applications of category theory, particularly in Haskell.

Music Genre

- Giving plenty of thought
- Music is often viewed as discrete
  - through keys, notes, named chords
  - as incorporated into scores
- But the physical sounds are waves with amplitude and frequency, associated with a pitch in Hz
- Chords have complex physical properties (harmonics), particularly when overtones are considered

Examples of Genre 1

- Popular music is the simplest form with generally no dissonance, all notes within a particular key, low range of pitch
- Classical music is much more varied than some people think:
  - Strays readily outside tonality (diatonic scales)
    - Chromaticism (foreign notes, not in diatonic key)
    - Use of chromatic 12-note scale throughout with only semitone intervals
    - Much dissonance
    - Individual experimentation: e.g. Ligati (every player has a different score), Chopin (rubato, expressive tempo)

Examples of Genre 2

- Jazz
  - Improvisation, some written score but much freedom of expression
- Film music
  - Links to drama, can be austere, disturbing
- Chanting
  - Very precise attention to the beat, crowd behaviour
- Microtones
  - Intervals less than a semitone are used
    - For freedom, easier on string instruments
    - For music from diverse cultures

Consequences

- The work presented here is suited to popular music, taking a simple discrete approach
- It can be readily adapted to the 12-note scale as still discrete
- However, did consider handling microtones, including Stockhausen's music, which is reputed to be amenable to category theory.
- This requires a move from discrete to continuous maths.

The Whitehead 'Now'

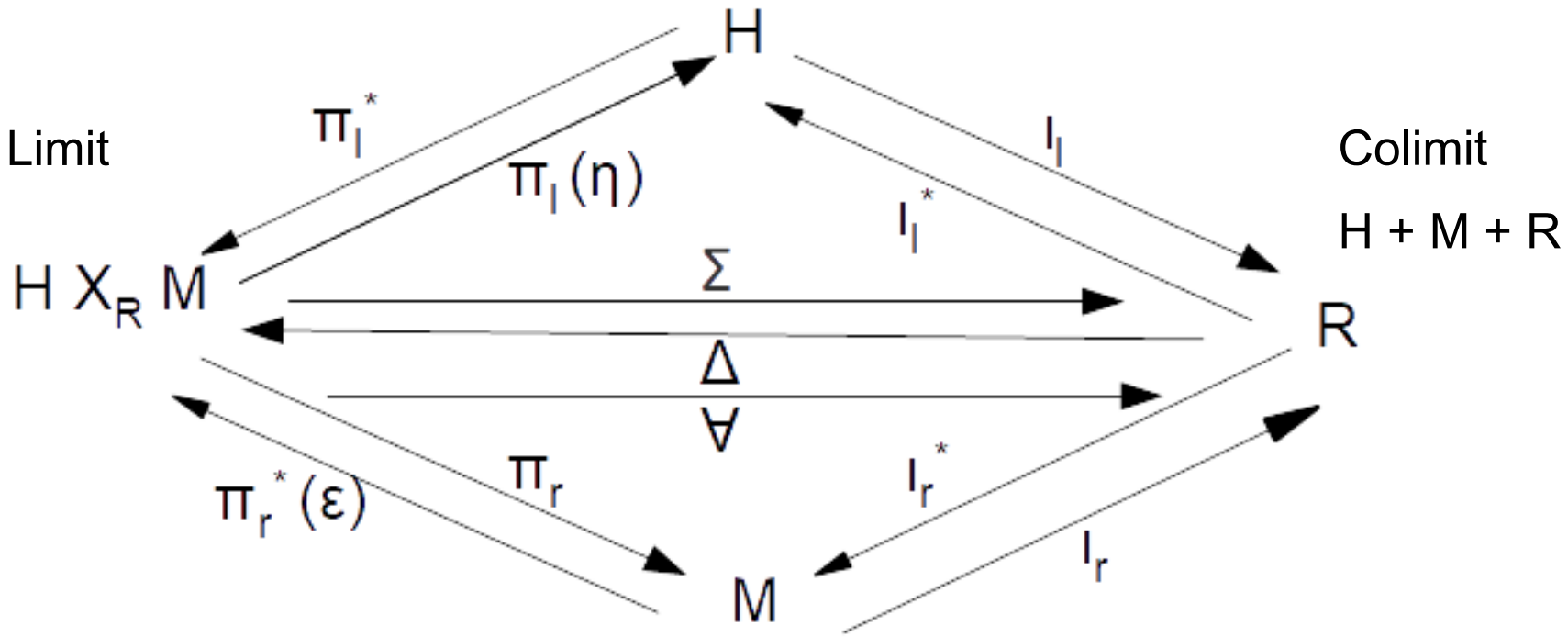
- How do physical sounds relate to the `Now' of the philosopher Alfred North Whitehead.
- Every entity in the World has a fleeting (covariant) capacity to act on and a (contravariant) perception to receive from its immediate surroundings.
- That is a local `now’ in both time and space giving rise to the synchronicity of the actual occasion where all the separate loci of each entity meet.
- These local `nows’ all compose to form the big NOW or OCCASION

Application of Category Theory to Popular Music

- Basic Categories:
  - Harmony H
  - Melody M
  - Rhythm R
- Organise in a topos as a pullback with:
  - Limit H X_R M
  - Colimit H + R + M

Dolittle Diagram

- Presented as a Dolittle Diagram with:
  - Adjointness between limit and colimit
  - Projection and inclusion arrows
- A Dolittle diagram is a pullback, which is also a pushout
  - Particularly useful for integrating intension (for example the score) and extension (the performance)
  - Intension/extension ideas are also from Aristotle



H X_R M: A Musical Performance is a Pullback of Harmony (H) along Melody (M) over Rhythm (R)

Pushout H + M + R	
Arrow	Purpose
Σ	creating the work
Δ	creativity of the work
$\forall$	genre of the work
$\pi_{\iota}(\eta)$	structural composition
π_{ι}^{*}	dominant harmony
π_{r}	distinct melodies
$\pi_{r}^{*}(\varepsilon)$	musical quality
I_{ι}	playing of ensemble
I_{ι}^{*}	identifying the harmony
I_{r}	playing each instrument
I_{r}^{*}	identifying each melody

Conclusion

- The Dolittle diagram is universal logic