Logical Types of Love

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Love in Physics

- Love is a word of art but also a word of science because of its relationship with Logic.

- Science never took off until Newton and his contemporaries showed how notions could be represented formally.

  - Physics has been very successfully advanced for such notions at first order but has not been able to advance into more advanced concepts like in biology and the humanities.

  - However current Process thinking recognises that all entities in the World are connected and therefore the subject matter of these more complex subjects must have boundary conditions that satisfy the logic of physics.
# Types of Love

<table>
<thead>
<tr>
<th>Greek</th>
<th>English</th>
<th>Latin</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>ἔρως [éros]</td>
<td>sexual passion</td>
<td>amor</td>
<td>loveliness of nature including the soul (Plato)</td>
</tr>
<tr>
<td>ἀγάπη</td>
<td>family love</td>
<td>amor</td>
<td></td>
</tr>
<tr>
<td>φιλία</td>
<td>friendship/affection</td>
<td>amicitia</td>
<td></td>
</tr>
<tr>
<td>στοργή</td>
<td>kinship</td>
<td>amor</td>
<td>+ patriotism</td>
</tr>
<tr>
<td>μανία</td>
<td>obsessive love</td>
<td>mania</td>
<td>+ jealousy, stalking</td>
</tr>
<tr>
<td>λυδός (Παιχνίδια)</td>
<td>playful love</td>
<td>ludus</td>
<td>+ a crush of young love</td>
</tr>
<tr>
<td>πράγμα</td>
<td>enduring love</td>
<td>pragma</td>
<td>+ commitment</td>
</tr>
<tr>
<td>Φιλαιτία [filaftía]</td>
<td>self love</td>
<td>amor sui</td>
<td>[selfishness]</td>
</tr>
<tr>
<td>φιλόσιτος</td>
<td>love of food</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ξενία.</td>
<td>feeling of hospitality</td>
<td></td>
<td>ξένος host/guest, foreigner, stranger (good/bad)</td>
</tr>
<tr>
<td>μεράκι</td>
<td>creative love</td>
<td></td>
<td>+ artistic</td>
</tr>
<tr>
<td>Φιλαδελία</td>
<td>brotherly/sisterly love</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ἀγαπητός</td>
<td>prized</td>
<td></td>
<td></td>
</tr>
<tr>
<td>φιλόστοργος</td>
<td>dearly loved</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ἀφθαρσία</td>
<td>eternal love</td>
<td></td>
<td></td>
</tr>
<tr>
<td>προσφιλής</td>
<td>agreeable pleasing</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Φιλαργυρία [filargyria]</td>
<td>love of money</td>
<td></td>
<td>[avarice]</td>
</tr>
<tr>
<td>ἀφιλάθαργος</td>
<td>unloving of public good</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Types of Love 2

- Compound words in Ancient Greek corresponds with composition in category theory. 200 examples in Oxford dictionary of synonyms for ‘love’

- Love takes many different forms but perhaps two main subdivisions:
  - Eros (romantic)
  - Plato

- A number of Platonic forms involve relations such as brother, mother.
Three Suggestions from Category Theory

1. Use of pullback/pushout constructions with tension between $X$ and $+\$

2. Use of Lawvere’s hyperdoctrine with adjointness between existential, diagonal and universal functors

3. Looking at lateralisation of processes in the brain and accompanying adjointness
Relationships in Category Theory

In category theory relationships can be represented by:

- Products (unqualified X, all possible pairs)
- Pullbacks (qualified X)
- Union (unqualified +, disjoint, duplicates not permitted)
- Pushouts (qualified +, direct sum)
Pullback of A and B in context of C

A $\times$ B is unqualified product

A $\times_C$ B is product in context of C

$f \circ \pi_l = g \circ \pi_r$
Pushout of A and B in context of C

If C is universal object *
sum is $A \cup B$ (disjoint union)

More general case
sum is $(A \cup B)/\sim$
$\sim$ is the equivalence relation for $f(c) \sim g(c)$
Quotient selection
Relationships 1

- Pullback is similar to relations in relational databases
  - Strong binding AND
  - An item together
  - Definitive pairing, in love context
    - A settled couple at a particular time
    - A married couple at a particular time
Pushout is similar to intersection in relational databases

- Weak AND
- A coexistence
- Temporary pairing, in love context
  - Have met in space and time
  - A dating couple
- ’Love’ is a particularly good example in the context of logic
  - to show how the different values (in the sense of a Heyting logic) on love are not necessarily monic
    - they may overlap
    - and not be independent one of another as elements in a Set.
Difference between Times and Plus

X is an item, pairing, partner (eros)
  – Limit is greatest lower bound
+ is a mixture, association (less committed, more platonic)
  – Colimit is least upper bound

Interplay between X and + plays a critical role in categorial applications

So we can do quite a lot with pullbacks and pushouts.

Can we be more expressive?

Not utilising pastings or multi-level architectures, freely available in category theory.
Example of Pasted Pullback - Music

Have 3 pullbacks: Pb2 X Pb1; Pb2; Pb1

C is category-object for Composer

Overall relationship is of Score with Variant and Composer in context of Occasion

Occasion is the Now of the philosopher AN Whitehead
Love in CT – Idea 2

• We can take the pullback/pushout idea to higher level:
  • a co-unit of adjunction with deconstructible subcategories
  • after Lawvere’s hyperdoctrine work in category theory
    • Bill Lawvere, Adjointness in Foundations, (TAC), Dialectica 23 (1969), 281-296
    • Non-classical Heyting intuitionistic logic
Pullback Diagram of Universe $S \times_{M+E} T$ in context of M+E

- Existential quantifier ($\exists$) is left adjoint to pullback (diagonal) functor ($\Delta$)
- Pullback functor ($\Delta$) is right adjoint to existential quantifier ($\exists$)
- Pullback functor ($\Delta$) is left adjoint to universal quantifier ($\forall$)
- Universal quantifier ($\forall$) is right adjoint to pullback functor ($\Delta$)

LCCC (Locally Cartesian Closed Category)
Heyting algebra (intuitionistic logic)
Pushout Diagram of Universe
$S +_{SX} T$ in context of $S_X$

There is tension between times ($S_X$) and plus ($S+$)
Dolittle Diagram for Universe

It is an adhesive category, also known as a pulation square, readily embedded in a topos

As \( \pi_{i} \) is monic, then so is \( I_{r} \): diagram is both a pullback and a pushout with limit, product, colimit, coproduct
Dolittle Diagram

- Presented as a Dolittle Diagram with:
  - Adjointness between limit and colimit
  - Projection and inclusion arrows
- A Dolittle diagram is a pullback, which is also a pushout
  - Particularly useful for integrating intension (for example the score) and extension (the performance)
  - Intension/extension ideas are also from Aristotle
Processes in the Brain

Use as an indication of

- Structures required
- Processes occurring
- Control of processes
Preliminary View from the Mind

With example of violin

- Left-hand performs pitch control (intonation)
- Right-hand performs bowing (articulation)

Hemisphere of brain

- Left-hand maps into rhs (intonation)
- Right-hand maps into lhs (articulation)

Coordination between lhs and rhs ('rhythm')

Control required as move through score

Musicians who learnt early in life have

- enhanced corpus callosum, to accommodate the control required of the two hemispheres in the brain
Co-ordination of the Brain in Category Theory

Need close (natural) relationship (adjointness) between the lhs and rhs

- The lhs will be an activity, a functor $A$ representing articulation
  - $A: \text{PERF} \rightarrow \text{PERF'}$
- The rhs will be an activity, a functor $I$, representing intonation
  - $I: \text{PERF'} \rightarrow \text{PERF}$
Monad - 3 cycles for left-hand violinist.

Hemisphere of Brain
Left

Corpus callosum

Right

Opposite situation for left-hand violinist.

A (functor) \rightarrow I (adjunction)
controlled by monad T (AI)

Intonation

Violin left-hand fingers

Process-thought

Articulation

Violin right-hand bow
Love in the Brain

Left-hand side is positive emotions (pleasure, P)
Right-hand side is negative emotions (fear, anxiety, A)

Adjunction PA or \(<P, A, \eta, \varepsilon>\)

P (Pleasure) is free functor (lhs of brain)
A (Anxiety) is underlying functor (rhs of brain)
\(\eta, \varepsilon\) are unit/counit of adjunction for offsets in mapping
Applications

- Always interested, as computer scientist, in computer applications of category theory, particularly in Haskell.
Published


Conclusions

- Category theory offers much sophistication in the way it handles relationships
  - Adjointness and monad
- Love is a particularly difficult concept to handle
  - The three areas tackled here:
    - X and +
    - Hyperdoctrine
    - Monad with adjointness
- Show its potential