Logical Types of Love

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Love in Physics

- Love is a word of art but also a word of science because of its relationship with Logic
- Science never took off until Newton and his contemporaries showed how notions could be represented formally.
  - Physics has been very successfully advanced for such notions at first order but has not been able to advance into more advanced concepts like in biology and the humanities.
  - However current Process thinking recognises that all entities in the World are connected and therefore the subject matter of these more complex subjects must have boundary conditions that satisfy the logic of physics.

Types of Love

Greek	English	Latin	Comment
ἔρως [éros]	sexual passion	amor	loveliness of nature including the soul (Plato)
ἀγάπη	family love	amor	
φιλία	friendship/affection	amicitia	
στοργή	kinship	amor	+ patriotism
μανία	obsessive love	mania	+jealousy, stalking
λυδός (Παιχνίδια)	playful love	ludus	+a crush of young love
πράγμα	enduring love	pragma	+commitment
Φιλαυτία [filaftía]	self love	amor sui	[selfishness]
φιλόσιτος	love of food		
ξενΐᾶ.	feeling of hospitality		ξένος host/guest, foreigner, stranger (good/bad)
μεράκι	creative love		+artistic
Φιλαδελία	brotherly/sisterly love		
άγαπητός	prized		
φιλόστοργος	dearly loved		
άφθαρσία	eternal love		
προσφιλής	agreeable pleasing		
Φιλαργυρία [filargyría]	love of money		[avarice]
άφιλάθαργος	unloving of public good		

Types of Love 2

- Compound words in Ancient Greek corresponds with composition in category theory. 200 examples in Oxford dictionary of synonyms for ‘love’
- Love takes many different forms but perhaps two main subdivisions:
    - Eros (romantic)
    - Plato
- A number of Platonic forms involve relations such as brother, mother.

Three Suggestions from Category Theory

1. Use of pullback/ pushout constructions with tension between X and +

2. Use of Lawvere's hyperdoctrine with adjointness between existential, diagonal and universal functors

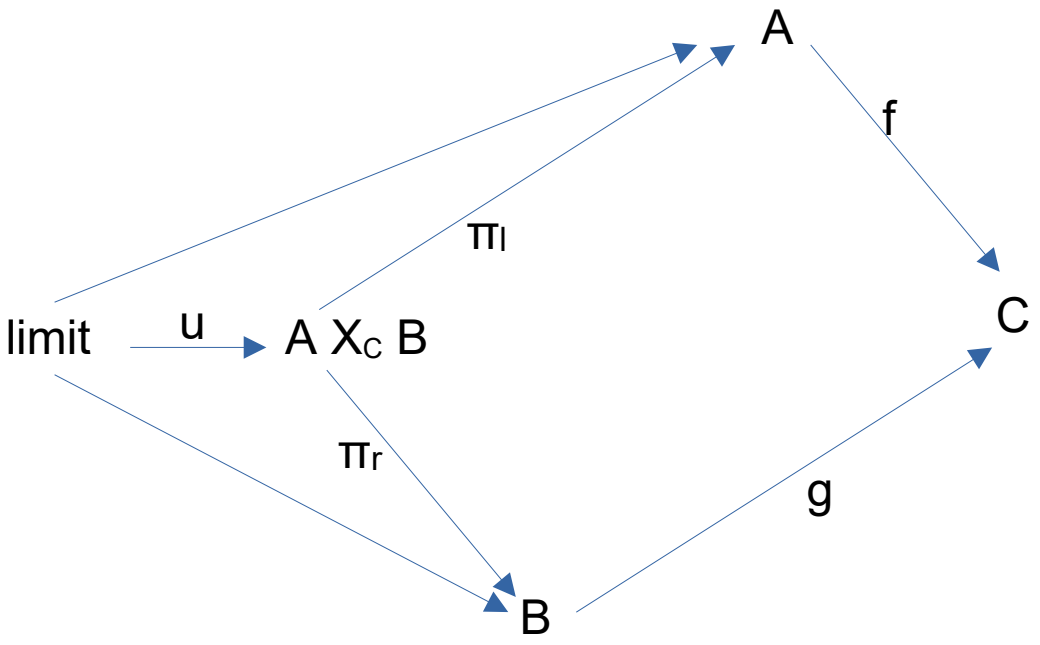
3. Looking at lateralisation of processes in the brain and accompanying adjointness

Relationships in Category Theory

In category theory relationships can be represented by:

- Products (unqualified X, all possible pairs)
- Pullbacks (qualified X)
- Union (unqualified +, disjoint, duplicates not permitted)
- Pushouts (qualified +, direct sum)

Pullback of A and B in context of C

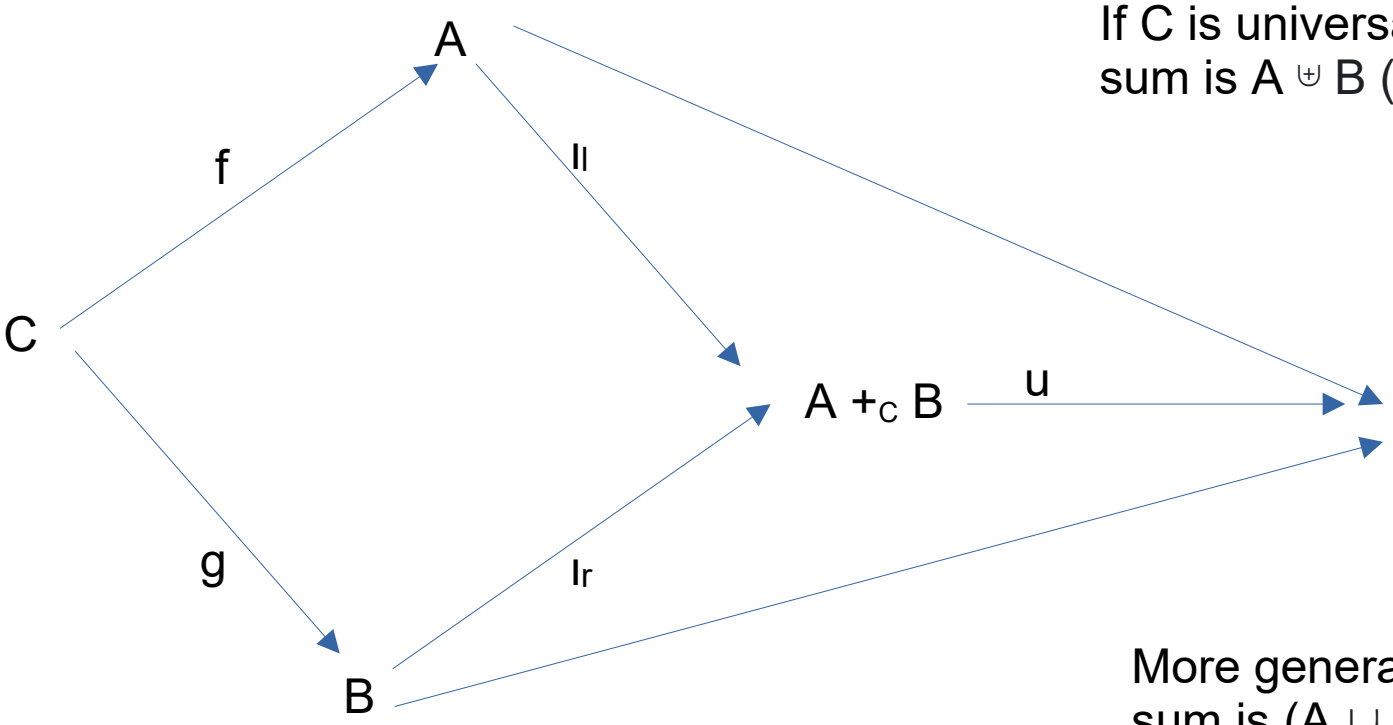


A X B is unqualified product

A X_C B is product in context of C

$f \circ \pi_l = g \circ \pi_r$

Pushout of A and B in context of C



If C is universal object *
sum is A ⊎ B (disjoint union)

More general case
sum is (A ∪ B)/~
~ is the equivalence relation for
f(c) ~ g(c)
Quotient selection

Relationships 1

- Pullback is similar to relations in relational databases
  - Strong binding AND
  - An item together
  - Definitive pairing, in love context
    - A settled couple at a particular time
    - A married couple at a particular time

Relationships 2

- Pushout is similar to intersection in relational databases
  - Weak AND
  - A coexistence
  - Temporary pairing, in love context
    - Have met in space and time
    - A dating couple
- Love' is a particularly good example in the context of logic
  - to show how the different values (in the sense of a Heyting logic) on love are not necessarily monic
    - they may overlap
    - and not be independent one of another as elements in a Set.

Difference between Times and Plus

X is an item, pairing, partner (eros)

- Limit is greatest lower bound

+ is a mixture, association (less committed, more platonic)

- Colimit is least upper bound

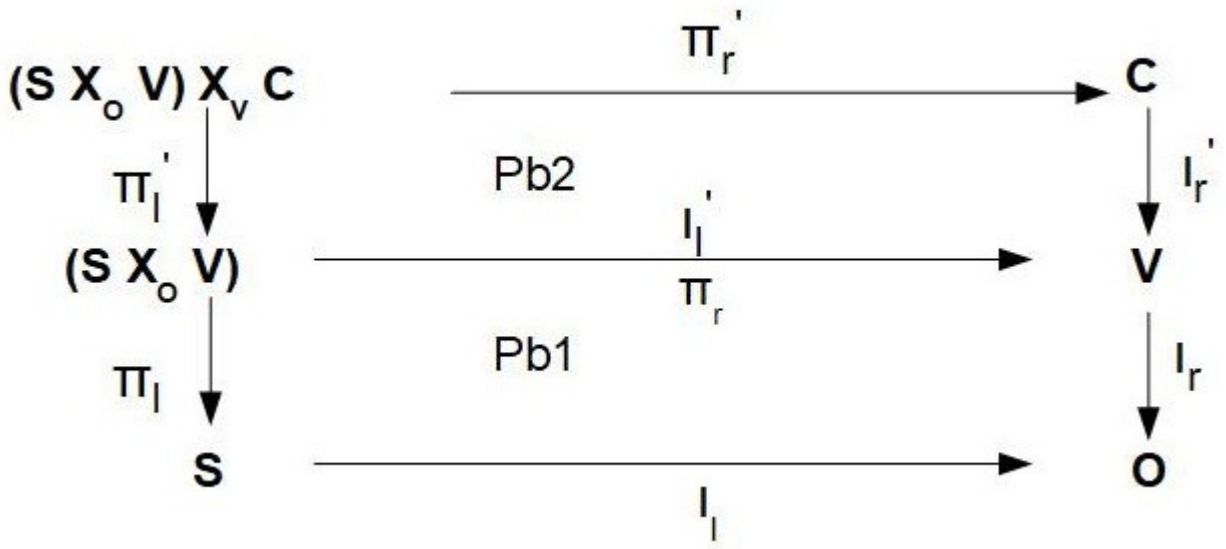
Interplay between X and + plays a critical role in categorial applications

So we can do quite a lot with pullbacks and pushouts.

Can we be more expressive?

Not utilising pastings or multi-level architectures, freely available in category theory.

Example of Pasted Pullback - Music



Have 3 pullbacks: Pb2 X Pb1; Pb2; Pb1

C is category-object for Composer

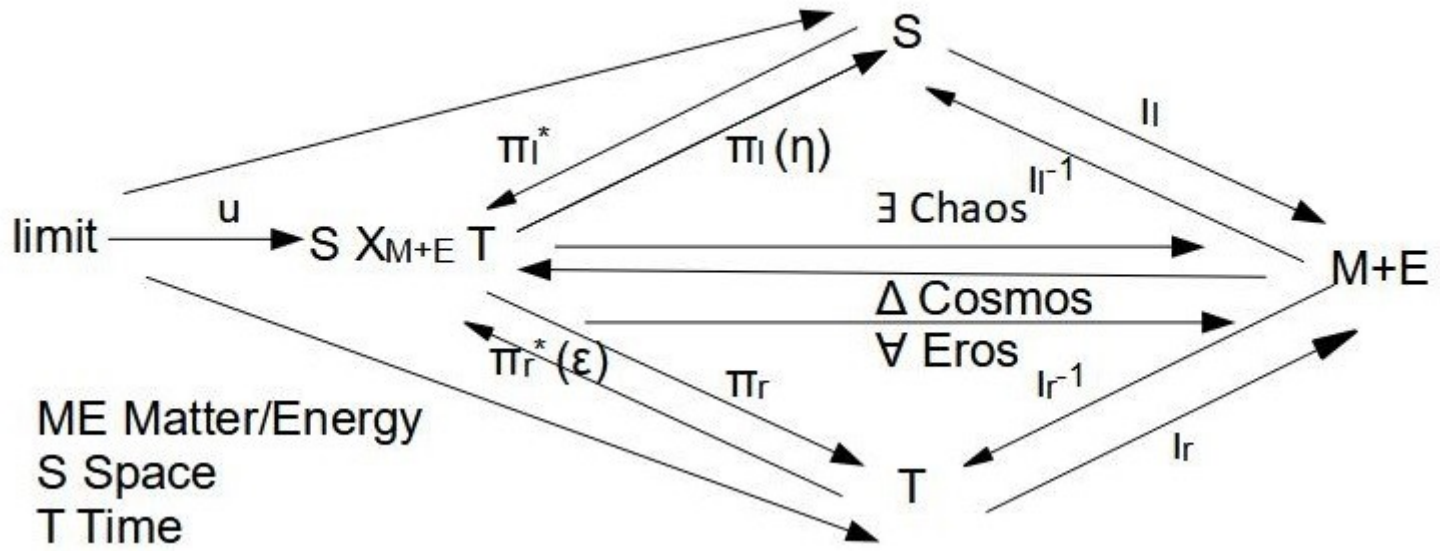
Overall relationship is of Score with Variant and Composer in context of Occasion

Occasion is the Now of the philosopher AN Whitehead

Love in CT – Idea 2

- We can take the pullback/pushout idea to higher level:
    - a co-unit of adjunction with deconstructible subcategories
    - after Lawvere's hyperdoctrine work in category theory
        - Bill Lawvere, Adjointness in Foundations, (TAC), Dialectica 23 (1969), 281-296
        - Non-classical Heyting intuituionistic logic

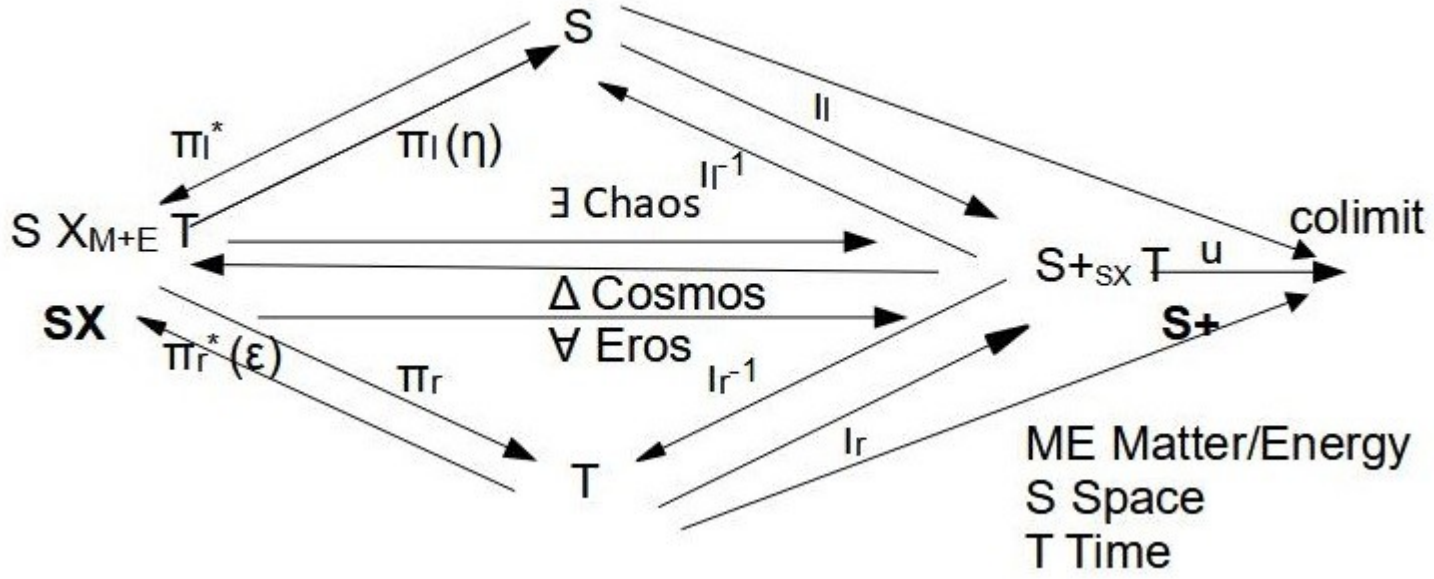
Pullback Diagram of Universe S X_{M+E} T in context of M+E



$$\exists \dashv \Delta \dashv \forall \qquad \pi_r^* \dashv \pi_r \qquad \pi_l^* \dashv \pi_l$$

Existential quantifier (∃) is left adjoint to pullback (diagonal) functor (Δ)
Pullback functor (Δ) is right adjoint to existential quantifier (∃)
Pullback functor (Δ) is left adjoint to universal quantifier (∀)
Universal quantifier (∀) is right adjoint to pullback functor (Δ)
LCCC (Locally Cartesian Closed Category)
Heyting algebra (intuitionistic logic)

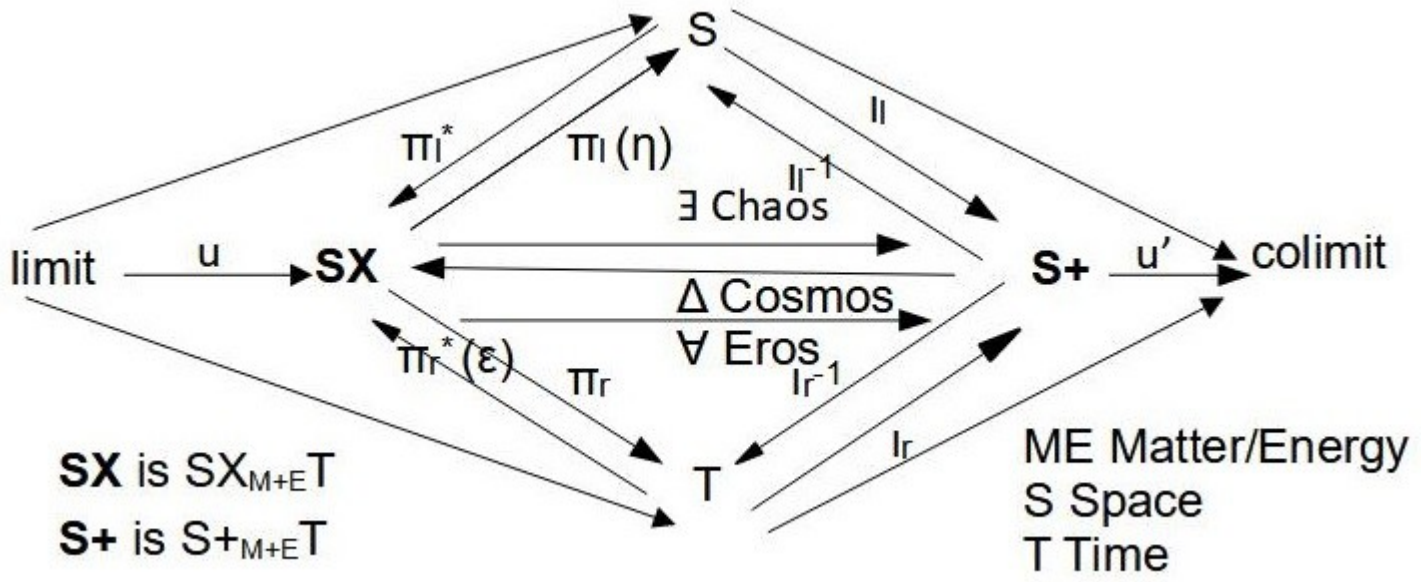
Pushout Diagram of Universe $S +_{SX} T$ in context of S_X



∃ ⊣ Δ ⊣ ∀

There is tension between times (SX) and plus (S+)

Dolittle Diagram for Universe



It is an adhesive category, also known as a pulation square, readily embedded in a topos

As π_l is monic, then so is l_r : diagram is both a pullback and a pushout
with limit, product, colimit, coproduct

Dolittle Diagram

- Presented as a Dolittle Diagram with:
  - Adjointness between limit and colimit
  - Projection and inclusion arrows
- A Dolittle diagram is a pullback, which is also a pushout
  - Particularly useful for integrating intension (for example the score) and extension (the performance)
  - Intension/extension ideas are also from Aristotle

Processes in the Brain

Use as an indication of

- Structures required
- Processes occurring
- Control of processes

Preliminary View from the Mind

With example of violin

- Left-hand performs pitch control (intonation)
- Right-hand performs bowing (articulation)

Hemisphere of brain

- Left-hand maps into rhs (intonation)
- Right-hand maps into lhs (articulation)

Coordination between lhs and rhs ('rhythm')

Control required as move through score

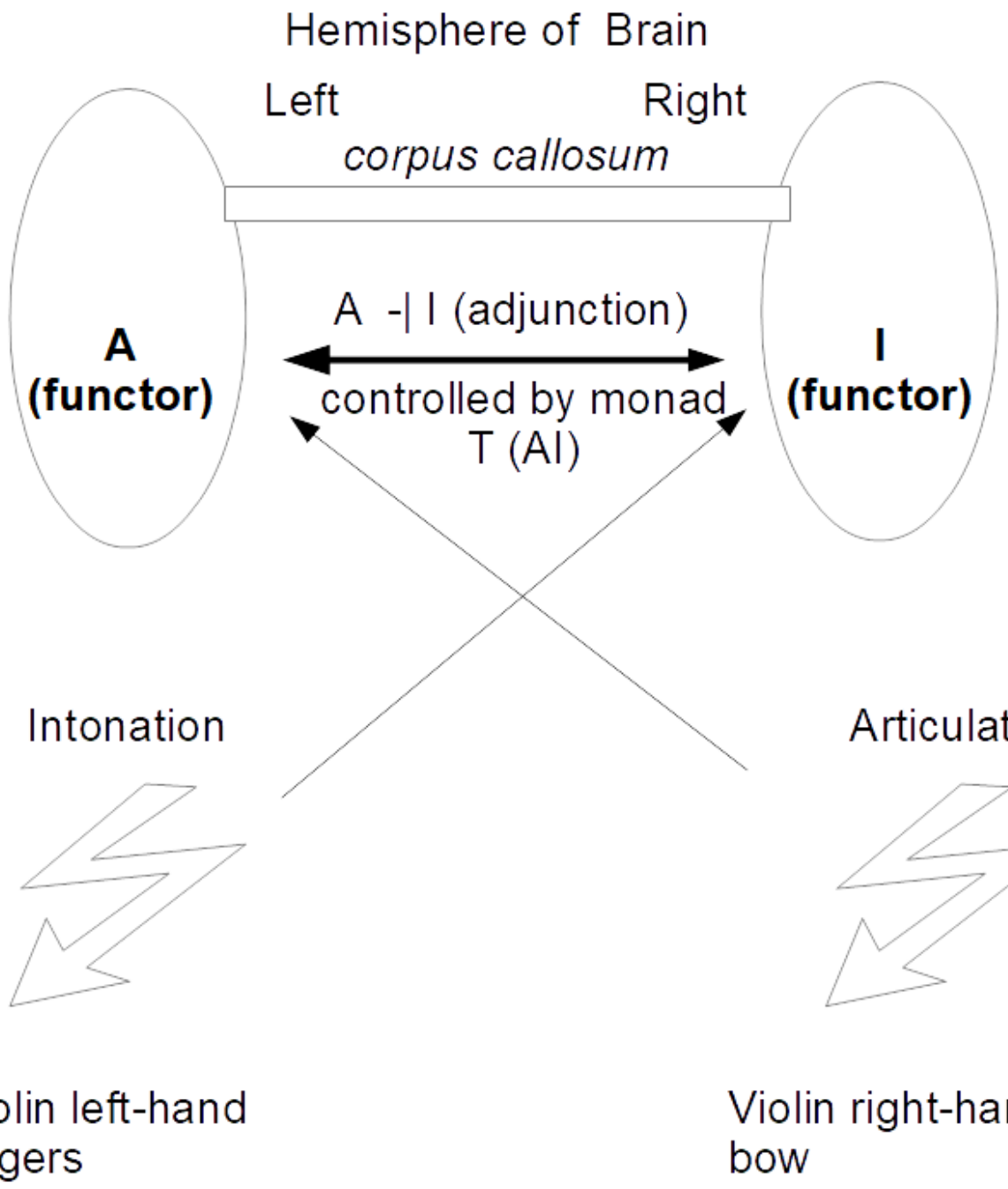
Musicians who learnt early in life have

- enhanced corpus callosum, to accommodate the control required of the two hemispheres in the brain

Co-ordination of the Brain in Category Theory

Need close (natural) relationship (adjointness) between the lhs and rhs

- The lhs will be an activity, a functor A representing articulation
  - A: PERF → PERF'
- The rhs will be an activity, a functor I, representing intonation
  - I: PERF' → PERF



Opposite situation for left-hand violinist

Monad - 3 cycles

Process-thought

Love in the Brain

Left-hand side is positive emotions (pleasure, P)

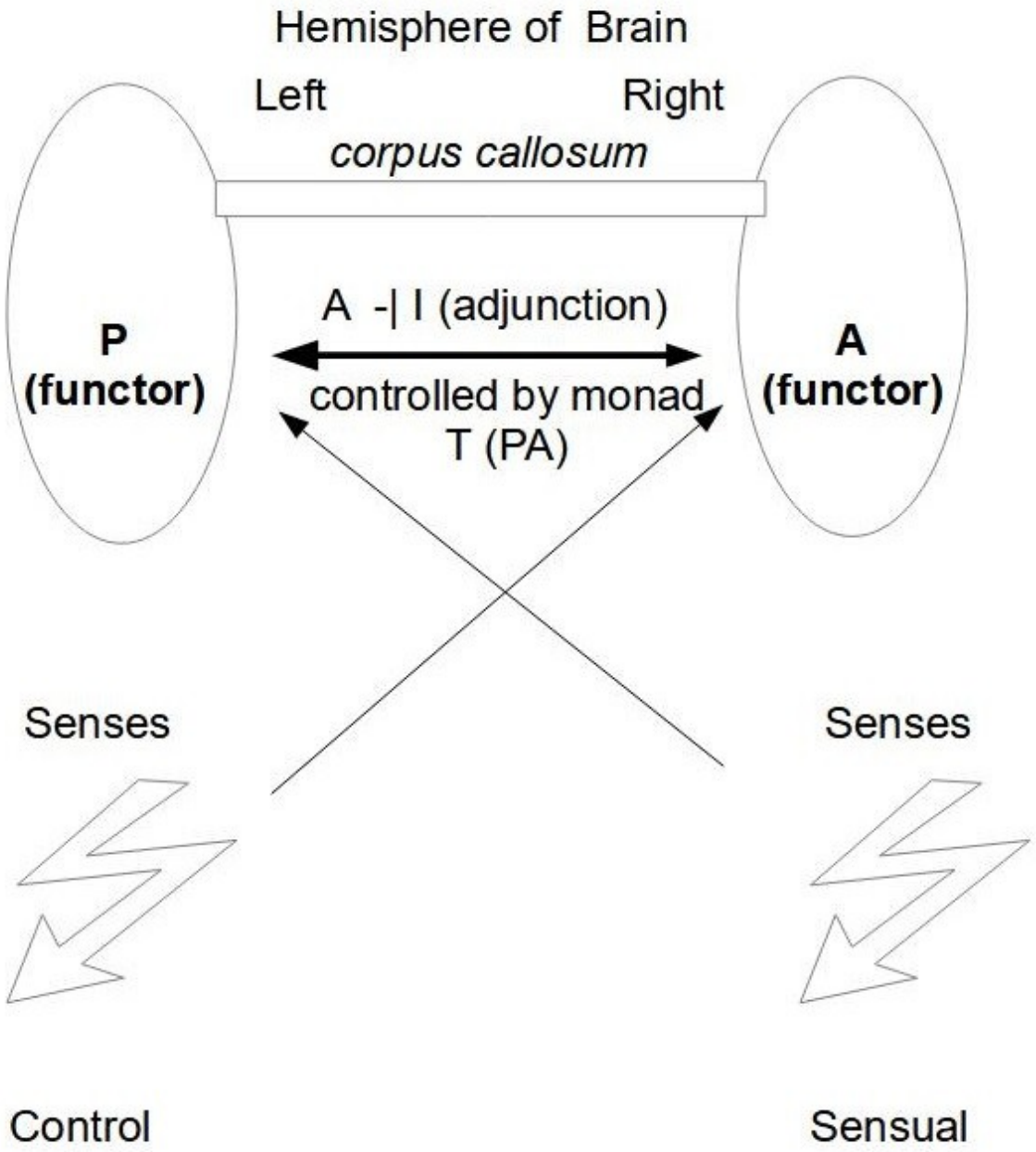
Right-hand side is negative emotions (fear, anxiety, A)

Adjunction PA or <P, A, η, ε>

P (Pleasure) is free functor (lhs of brain)

A (Anxiety) is underlying functor (rhs of brain)

η, ε are unit/counit of adjunction for offsets in mapping



Process-thought

Applications

- Always interested, as computer scientist, in computer applications of category theory, particularly in Haskell.

Published

- Rossiter, Nick, & Heather, Michael, Musical Performance: a Composition of Monads, Sociology and Anthropology 7(4), 178-188 (2019)
- Rossiter, Nick, & Heather, Michael, Physical Sounds as Colimits in the Topos under Monad Control, ANPA 40, Aporia, 40th Anniversary Proceedings 1979-2019, John Ceres Amson (ed), 541-578 (2020)

Conclusions

- Category theory offers much sophistication in the way it handles relationships
  - Adjointness and monad
- Love is a particularly difficult concept to handle
  - The three areas tackled here:
    - X and +
    - Hyperdoctrine
    - Monad with adjointness
  - Show its potential