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# Really Really Real Numbers

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## ***Abstract***

Number is a three level structure of which the natural numbers are only a first order model. At the third level of the Universe are the numbers one, the exponential and  $\pi$ .

## **The Natural numbers are not natural**

Nature has no human numbers, no right angles nor straight lines. Is Wigner's enormous usefulness of mathematics really 'something bordering on the mysterious' with no rational explanation? Is Kronecker right to claim all numbers are of human invention except integers and zero? ANPA from its beginning has been fascinated by the integers of which a combinatorial hierarchy may be able to provide values for both the fine structure constant and that of Newton's Universal Gravitational with the Parker-Rhodes conjecture within 0.03% of the known experimental value. However a reason for any Wigner coincidence awaits explanation but so does that for most significant number theory. Numbers known as the 'Reals' are hardly real except in the mind but rather first order models relying on reductionist assumptions and for convenience will be referred here to as 'ordinary numbers'. These cannot be reliably applied to higher order phenomena where most of today's problems lie. Arithmetic is always first order, algebra is only generalised arithmetic, a topology is no more than the family of open subsets of some set that attempts to model higher order although still inherently first order. The same characterises more sophisticated numbers like Tensors, Surreals, Hilbert Spaces or Einstein's field equations— they are all only valid to first order. For this reason Alfred North Whitehead swapped the set theory of his Principia for the comprehensiveness of metaphysics. Current mainstream science however still seems stuck in the former.

Early ANPA followed 20th Century science in treating the irreal as isomorphic to sets. From which it follows that the number 'two' exists as an exact integer in Nature as evidenced by Newton's inverse square law. But Whitehead had by the 1920's moved to Process as the controlling force of the Universe. Ted Bastin one of the founders of ANPA advocated Process at that time but while still adhering to Newton. Whitehead on the other hand having long abandoned Newton embarked on a path debunking the latter's three Laws of Motion. Whitehead's Universe is an occasion of events not a Euclidean receptacle. Even current mainstream cosmologists take the view that the Universe departs from the Euclidean by about 2% they estimate<sup>1</sup>. However this figure may be too low as it is more likely to be related to the value of the Euler number 'e'.

One very important principle of ANPA is its 'primary purpose' set out at the start of its Statement of Purpose: 'to consider coherent models based on a minimal number of assumptions...' The problem is that any presumption destroys the basis of objective inference. We prefer here not to rely on any assumption but only on the empiricism of physics as perceived by the senses. Whether we have succeeded we have to leave to the reader. The difficulty is that the use of any assumption means that the outcome of a line of reasoning may have originated from the assumption leading to the Anti-Foundation Axiom<sup>2</sup> which Russell called an impredicative definition, and Hermann Weyl a vicious circle. An example of this vicious circle is to be found in the doctoral thesis of the late Stephen Hawking which is happily available on line<sup>3</sup>. From the beginning he assumes the existence of irreal like zero and infinity and not surprisingly that leads to a belief in the Big Bang from postulated zero and Black Holes from postulated infinity. As it happens Category Theory comes to the rescue of Black Holes as a free functor valued category but it cannot support the Big Bang. Empirically nowhere in the Universe can be found 'nothing'.

Euclidean Space is not connected naturally: for the elements of a set are independent one of another. Some arbitrary functions like the Peano conjectures can be applied to relate them at will but the Universe is inherently not arbitrarily connected but strictly related by a higher order relationship. In order to represent this relationship formally it is necessary to rise above sets to Category Theory which in its pure form is a metaphysical language as opposed to the modelling language of Set Theory. This problem with Set Theory is not only that the elements of a set are

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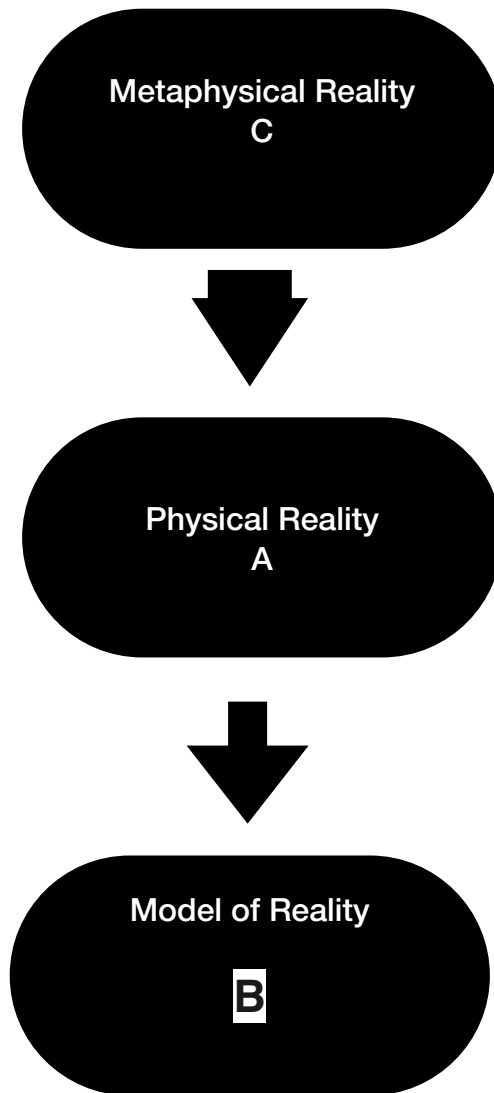
<sup>1</sup>John Barrow <https://youtu.be/hha2295o3CM> at 39:35 min.

<sup>2</sup> M. Rathjen, PREDICATIVITY, CIRCULARITY, AND ANTI-FOUNDATION, <https://www1.maths.leeds.ac.uk/~rathjen/russelle.pdf>

<sup>3</sup> <https://www.docdroid.net/VBP05Hq/pr-phd-05437-cudl2017-reduced-pdf#page=15>

independent but also a set may not be a member of itself. Whereas a self reflexive category can be one of its own objects.

**Ordinary Numbers are only on the lowest rung of three Realities.**



Dear Lyn The entities in the Universe are non-separable<sup>4</sup> and are all therefore relatable. Those relations are all relatable and then these relations

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<sup>4</sup>Whitehead, A.N. (1919), An Enquiry on the Principles of Natural Knowledge ,Cambridge University Press, Cambridge; Whitehead, A.N. (1920), The Concept of Nature ,Cambridge University Press, Cambridge; Whitehead, A.N. (1922),

are relatable. There builds up a Universe of possible relations of relations of relations at which closure kicks in but this three level relationship can then start again.

As a collection of categories the Universe may therefore be viewed as a staircase of three rung processes interlocked by adjointness as a down from the meta-meta level. These are continuously related downwards exponentially.

This downward exponential process  $\mathbf{A}^{\mathbf{C}} \rightarrow \mathbf{B}^{\mathbf{A}}$  is under the control of the three level Heyting Logic

$$C \times A \leq B$$

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$$C \leq A \Rightarrow B$$

This may be interpreted: **A** integral with its context **C** (such as the rest of the Universe) precedes **B**. That is mutually and conversely true with the requirement that **C** precedes the inference **A** implies **B**.

Formally this is the inherent internal language of a pure Topos that has exponential objects but no ordinary numbers. That is empirically the same as the Universe. The pure Topos is to be distinguished from its earlier Grothendieck version whose logic relied on set theory. The Grothendieck in mainstream pure mathematics is therefore but a Boolean model of a pure topos.

Ordinary numbers all belong in the lowest rung as axiomatic models. Whitehead insists that a system of axioms needs to be independent, consistent and be proved to exist. Peano needs around a dozen axioms to establish arithmetic. Zero is assumed as natural but never a successor number. Closure is also assumed under the equality relations (reflexive, symmetry, transitive) and the injective successor function by induction but there are issues over the mix between first and second order. Again the integers are make-believe only existing in the mind and not to be found in physics. Although claimed 'real' the ordinary numbers include the integers, the rationals and the irrationals all of which can be represented by a point on the straight line. There is still just that little problem that there are no such thing anywhere as straight lines other than in the mind.

Innate connectivity in metaphysics imposes a comprehensive relationship between entities (represented by the arrow in Category Theory) covering all

possible interpretations. One such effect is that every entity may be thought of as a first order model of every other entity and no model can be treated as wholly wrong. This belongs to Whitehead's 'every truth is a partial truth'<sup>5</sup>. As empirically there are no two identical entities in the World the use of the equals sign '=' is a sleight of hand. Because all is process Whitehead claims that 'two times three equals six' should be read as 'two times three becomes six'. Clearly the left and right hand sides of the equation

$$2 \times 3 = 6$$

are not the same. There is some assumed interpretation. This is one reason why Whitehead & Russell in their Principia had such difficulty in proving<sup>6</sup>

$$1 + 1 = 2$$

The point being: how do we know that each of the two 'ones' have the same definition? Do any numbers exist? Possibilities would still appear to be

$$0, 1, \sqrt{-1}(i), e, \pi, \infty$$

## Ordinary Number Zero

A special case is the number zero which is a mathematical or mythical creation as a cardinal number and again not to be found in the physical universe and therefore not prove to exist. The arithmetical Zero has been invented many times throughout the World because of the need for a place-holder in any consecutive number system. The algebraic zero on the other hand was invented by the early English polymath Thomas Harriott in the 16th century. He had some doubts about it because like all equations in physics there is always some issue whether the terms are pure numbers or have implied dimensions. To be safe Harriott would write a quadratic equation as

$$ax^2 + bx + c = 0, 0, 0.$$

but realised that it was really an ordering for which he invented the  $\leq$  sign although it does not help to relate the cardinals and the ordinals which have no natural correspondence.

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<sup>5</sup> Alfred North Whitehead, Dialogues, 1954: Prologue.

<sup>6</sup> Alfred North Whitehead & Bertrand Russell Principia Mathematica Vol I Part II 328-383 attempts to define the cardinal numbers 1 and 2 without success. Volume II devotes 724 pages in an unsuccessful attempt to formalize the arithmetic axioms of Peano and fails to establish the fundamental  $1+1 = 2$ .

Zero has difficulties which are often swept under the carpet. The isomorphism with the null set is anomalous. Compare the difficulties with null data values in databases. By the Closed World Assumption (that is the CWA) of relational databases any tuple not in the relation represents a false proposition. However this does not guarantee integrity of the tuples that are true<sup>7</sup>. In particular the handling of nulls poses many semantic problems as while they are treated as data values, there are different interpretations such as missing-but-applicable and missing-and-inapplicable. Nulls therefore make a system undecidable. Some more recent experimental versions of the relational model do not permit nulls, for example RAQUEL developed at Northumbria University.

The Council of the Royal Statistical Society has recently raised queries about the Null hypothesis in significance testing.<sup>8</sup> Even more recently this year the Royal Astronomical Society has announced doubts about the zero point constant of the bolometric correction scale which is the basis for relative luminosity in astrophysics.<sup>9</sup> This has far reaching effects on reported work for 80 years on luminosity with knock on doubts for understanding topics like the accelerating expansion of the Universe and general cosmology.

## Numbers minus One and Infinity

Both the numbers minus one and infinity belong in the lowest rung of reality. The numeral -1 only provides a label to distinguish positive and negative numbers but its imaginary root 'i' goes further as an operator to express imaginary numbers in general but cannot be said to be truly a natural number. Likewise infinity ' $\infty$ ' is a model in mathematics and does not exist anywhere in that form in physics. For 50 years, mathematicians have believed that the total number of real numbers is unknowable<sup>10</sup>. However the corresponding concept of what Whitehead calls 'unbounded' is available in Category Theory as a free functor valued category and able to cope with the open closure of the Universe which is not possible with ordinary numbers.

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<sup>7</sup>Heather, Michael, Livingstone, David, & Rossiter, Nick, Logical Foundations for the Infrastructure of the Information Market, I-ESA, Berlin, 25-28 March, published in Enterprise Interoperability III, New Challenges and Industrial Approaches, Mertins, Kai, Ruggaber, Rainer, Popplewell, Keith and Xu, Xiaofei, (edd) 624-637 Springer (2008).

<sup>8</sup> Ovens, Matthew. "The Logic of Null Hypothesis Significance Testing". Retrieved from [YourStatsGuru](http://YourStatsGuru).

<sup>9</sup> Z Eker, V Bakış, F Soyduğan, S Bilir, On the zero point constant of the bolometric correction scale. Monthly Notices of the Royal Astronomical Society, 503, (3) May 2021, 4231–4241, (May 2021) <https://doi.org/10.1093/mnras/stab684> <https://blog.oup.com/authors/zeki-eker/>

<sup>10</sup> <https://www.quantamagazine.org/how-many-numbers-exist-infinity-proof-moves-math-closer-to-an-answer-20210715/>

Newton (Euclidean), Einstein (static  $\Lambda$  gravity balance), Hubble (expanding), de Sitter (matterless acceleration), Friedman (matter in motion), Edington-Lemaître (infinite after ten + four accelerating billion years), Tolman (oscillating and 'kinky' inhomogeneous), D'Albe & Charlier (fractal), Kasner (Googleplex 10100), Dirac (gravity decaying), Einstein-Rosen (undulating), Milne (no expansion nor recession), Strauss (swish-cheese), Landau & Lifshitz (perturbed), Schrödinger (particle to wave), Gödel (recurring), Holmberg (table-top), Bondi & Lyttleton (electric), Gamow (hot), von Weizsäcker & al (turbulent), Bianchi & Taub (expanding spaces I-IX), Wilkinson & Partridge (smooth), Misner chaotic), Misner (mix in key), Thorne & al (magnetic), Brans & Dicke (gravitational weakening), Alfvén & Klein (antimatter), Hawking & Ellis (a singularity), No Success (cold & tepid), Politzer & al (high energy particles), Georgi & Glashow (Grand Unified Theory), Dirac (magnetic monopole), Guth (inflationary), NASA Satellites (chaotic inflationary), Current Mainstream (self-reproducing eternal inflationary), Post-modern (random, probable, anthropic, possible, home-made, naturally selected, fake, with nothing original, Boltzmann's, wrap-around, quantum, self-creating, colliding, light dying, hyper-universes, best-buy, the preposterous, the puzzling).

### **Panel of possible types of Universes**

On the other hand the number 'one' is at the top of the three level ladder as a 'really, really real number'. How many true 'ones' are there in the Universe? Because of non-severability there is no 'one' of anything. There is therefore only one 'one' of the Universe, namely the Universe itself. We use the self-reflective image of that 'Universal one' to carve out and designate 'one of anything' in the World.

### **The Top rung really really real numbers '1', 'e' and ' $\pi$ '**

This use of 'one' is viable because the 'really really real' number 'e' relates objects recursively top-down. It should not be surprising therefore that the 'e' is so fundamental to be needed in many physics energy equations where again it self-reflects down examples of mass/energy in the Universe.

Of course the 'really really real' pièce de resistance is the number  $\pi$  which is profligate within nature to be found: in so many equations for geometry with curvature such as infinite series & products, and integration in mathematics; in physics such from the simple pendulum to Einstein's field equations, the cosmological constant, Heisenberg's uncertainty principle, electromagnetism, Kepler's Laws of Planetary Motion, etc.

Can we say we understand anything of fundamental science unless we can explain the ubiquity of  $\pi$ ? Here is a suggestion. It is not too well known that there is no equation for the perimeter of an ellipse. It cannot be accurately calculated or drawn in two or three dimensions. If you're clever you can calculate it approximately. The closest seems to have been evaluated by the prodigy Indian mathematician Ramanujan (1887- 1920)<sup>11</sup>

## **Examples of First Order Methods relying on Number**

First Order methods have been extremely successful for three centuries in dealing with local systems but become unnecessarily complicated on attempts to extend them at higher order like for global systems. This problem applies to swathes of mainstream science: The limitations of that bottom level in science should be reviewed as it is blinkered.

For example the common view is that the current Universe was initiated at the Bing Bang by quantum fluctuations — that seems to require quantum mechanics to have preceded the Universe! It has then continued up to today by some process of evolution. This is all posited firmly on the existence of Peano's number system. However despite this widespread view there abound many possible conflicting views of detail. These are briefly summed up with their source in the Panel<sup>12</sup>. We have only space just to list other topics in Physics constrained to first order with these red scientific health warnings:

### **I. Quantum Mechanics elevated to Quantum Theory**

The early originators only had ordinary numbers like the linear differential Schrödinger equation to use as a model to form a misleading stable system whereas the higher order topos shows it's the other way round with the quantum world exact.

### **II. Quantum Computing**

Quantum logic followed the first order classical model of ordinary numbers with the 'quantum bit' instead of the higher order 'quantum monad'. The only progress is with local binary successes at very low temperature in the

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<sup>11</sup> Paul Abbott, On the Perimeter of an Ellipse, *Mathematica Journal* 11 (2) 172-185 (2009): <https://www.mathematica-journal.com/2009/11/23/on-the-perimeter-of-an-ellipse/>

<sup>12</sup>From John D Barrow, *The Book of Universes: Exploring the Limits of the Cosmos*, Norton, (2013).



region of superfluity and super conductivity<sup>13</sup>. For room temperature a strange XZ Calculus<sup>14</sup> with Boolean spiders has been devised from monoids rather than monads and with bottom-up generators rather than metaphysical top-down process

### **III. Statistics and Probability**

First order statistical models can be way out when applied to higher order phenomena. Compare then grossly wrong predictions for the COVID-19 virus<sup>15</sup>. 'Probability' is not really undefined other than as a relationship between ordinary numbers like 'the chance of throwing a six with dice is one sixth'. It is a principle found in the Universe only at first order. Attempts at higher order as a quantum physics reality can still only be a first order model.

### **IV. Newton & Gravity**

As mentioned above Newton's inverse square just describes Euclidean geometry not gravitation. Whitehead dismisses: Newton's First Law of Motion because no straight line exists for an entity to continue along; and the Second Law as a circular argument that the force of mass times acceleration just equals force times acceleration. Newton's Third Law of Reaction needs to be recast with higher order Heyting logic.

### **V. General & Special Relativity**

Einstein's powerful theories are neither right nor wrong but need to be recognised as only first order being derived from ordinary numbers

### **VI. Chaos Theory**

Mainstream treats first order chaos as disorganised ordinary numbers rather than a higher order phenomenon.

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<sup>13</sup> Narasimhachar, V., Gour, G. Low-temperature thermodynamics with quantum coherence. Nat Commune 6, 7689 (2015). <https://doi.org/10.1038/ncomms8689>

<sup>14</sup> Heunen, Chris; Vicary, Jamie (2019). Categories for Quantum Theory. Oxford University Press. doi:10.1093/oso/9780198739623.001.0001. ISBN 9780198739616.

<sup>15</sup> SAGE, Considerations in implementing long-term 'baseline' Non- Pharmaceutical Interventions (NPIs) [https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment\\_data/file/992738/](https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment_data/file/992738/)

## VII. Pure Mathematics & Symbolic Logic

Mainstream is restricted mainly to local first order ordinary numbers whereas most problems today are at a higher order global level. Now it is being gradually recognised that much public finance is being wasted on useless 'symbol-bashing' of ordinary numbers. Leicester University has therefore recently closed down research in pure mathematics<sup>16</sup>.

## VIII. Information Technology & AI

Both are currently limited by the restrictions of the Von Neumann machine architecture which is no more than a register that counts ordinary numbers up and down<sup>17</sup>

## IX. Climate change

Mainstream research on climate change is limited to first order ordinary numbers modelling the effect of human activity on the atmosphere which neglects the top level of natural processes of geophysics.

## X. Thermodynamics

The Laws of Thermodynamics are entirely axiomatic relying on ordinary numbers and are anomalous

## Conclusions

To sum up the really really real numbers are not Boolean Cardinals but Heyting 1, 2 (namely 'e') & 3 (namely ' $\pi$ ') and reside at the third level of abstraction of metaphysics in contrast to the first order home of the ordinary so-called 'real' numbers. These need to be understood in the context of the pure topos where concepts like Whitehead's *rect* replace the straight line and the *punct* is his analogue of a geometric point

## Final question

Could the crack between the Topian and the Euclidean Universe be whence cometh life?

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<sup>16</sup><https://cgowers.wordpress.com/2021/01/30/leicester-mathematics-under-threat-again/>

<sup>16</sup> Rossiter, B N, Heather, M A, & Sisiaridis, D, Process as a World Transaction, Proceedings ANPA 27 Conceptions, 122-157 (2006). pdf [Applying Monads to Distributed Database Transactions]

