Really Really Real Numbers

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Talk to ANPA 2021 Meeting by Zoom
12-14 August 2021

Outline of Talk

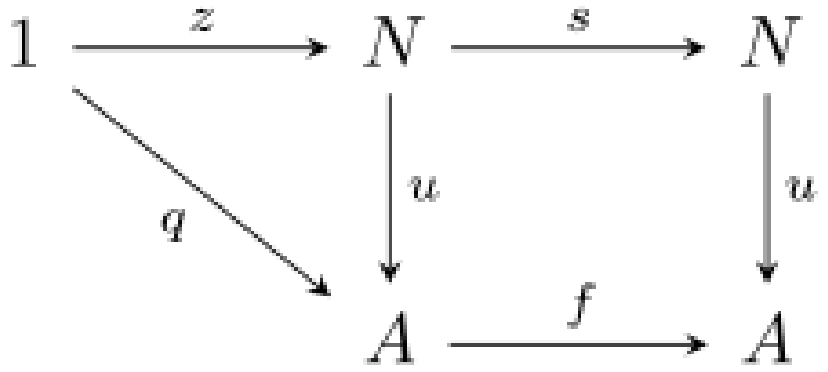
- The Natural Numbers are not natural
- Ordinary Numbers are only on the lowest rung of three Realities
- Ordinary Number Zero
- Numbers Minus One and Infinity
- The top rung Really Really Real Numbers ‘1’, ‘e’ and ‘π’
- Examples of First Order Methods relying on Number

The Natural Numbers 1

- Natural is defined by counts of sheep in fields in some texts
  - Excludes zero in some quarters but can you not have an empty field with 0 sheep
  - In computing science, natural numbers are the integers counting from 0 upwards
- Peano's axioms for generating infinite sets
- More acceptable is defining the natural numbers by arrows
    - The successor function with f_0=a, then f_1=s(a), f_2=s(s(a)), ...

The Natural Numbers 2

- The successor function is employed (as recursion) in the Natural Numbers Object (NNO) of category theory (Lawvere):



N, A are objects in category E, a topos; 1 is the terminal object;
s is the successor function

u z = q; f u = u s

A natural numbers object (NNO) in a topos is an object that behaves in that topos like the set N of natural numbers does in Set

Looks like Categorification

Problems with Numbers 1

- Is Wigner's enormous usefulness of mathematics really 'something bordering on the mysterious' with no rational explanation?
- Was Kronecker right to claim all numbers are of human invention except integers and zero?
  - Interested in finitistic mathematics, early lead-in to intuitionistic approaches
- ANPA from its beginning has been fascinated by the integers of which a combinatorial hierarchy may be able to provide values for both the fine structure constant and that of Newton's Universal Gravitational with the Parker-Rhodes conjecture within 0.03% of the known experimental value
  - However a reason for any Wigner coincidence awaits explanation but so does that for most significant number theory

Problems with Numbers 2

- Numbers known as the ‘Reals’ are hardly real except in the mind but rather first order models relying on reductionist assumptions and for convenience will be referred here to as ‘ordinary numbers’
- These cannot be reliably applied to higher order phenomena where most of today’s problems lie
  - arithmetic is always first order
  - algebra is only generalised arithmetic
  - a topology is no more than the family of open subsets of some set that attempts to model higher order although still inherently first order
  - the same characterises more sophisticated numbers like Tensors, Surreals, Hilbert Spaces or Einstein’s field equations
- They are all only valid to first order

Problems with Numbers 3

- For this reason Alfred North Whitehead swapped the set theory of his Principia for the comprehensiveness of metaphysics
- Current mainstream science however still seems stuck in the former

Whitehead's Process

- Early ANPA followed 20th Century science in treating the irreals as isomorphic to sets
  - From which it follows that the number ‘two’ exists as an exact integer in Nature as evidenced by Newton’s inverse square law
- But Whitehead had by the 1920s moved to Process as the controlling force of the Universe
- Ted Bastin one of the founders of ANPA advocated Process at that time but while still adhering to Newton

Whitehead's Process 2

- Whitehead on the other hand having long abandoned Newton embarked on a path debunking the latter's three Laws of Motion
  - Whitehead's Universe is an occasion of events not a Euclidean receptacle
- Even current mainstream cosmologists take the view that the Universe departs from the Euclidean by about 2% they estimate
- However this figure may be too low as it is more likely to be related to the value of the Euler number 'e'

Assumptions

- One very important principle of ANPA is its ‘primary purpose’ set out at the start of its Statement of Purpose: ‘to consider coherent models based on a minimal number of assumptions…’
- The problem is that any presumption destroys the basis of objective inference. We prefer here not to rely on any assumption but only on the empiricism of physics as perceived by the senses. Whether we have succeeded we have to leave to the reader
- The difficulty is that the use of any assumption means that the outcome of a line of reasoning may have originated from the assumption leading to the Anti-Foundation Axiom which Russell called an impredicative definition, and Hermann Weyl a vicious circle
- An example of this vicious circle is to be found in the doctoral thesis of the late Stephen Hawking. From the beginning he assumes the existence of irreals like zero and infinity and not surprisingly that leads to a belief in the Big Bang from postulated zero and Black Holes from postulated infinity
- As it happens Category Theory comes to the rescue of Black Holes as a free functor valued category but it cannot support the Big Bang. Empirically nowhere in the Universe can be found ‘nothing’

Connections

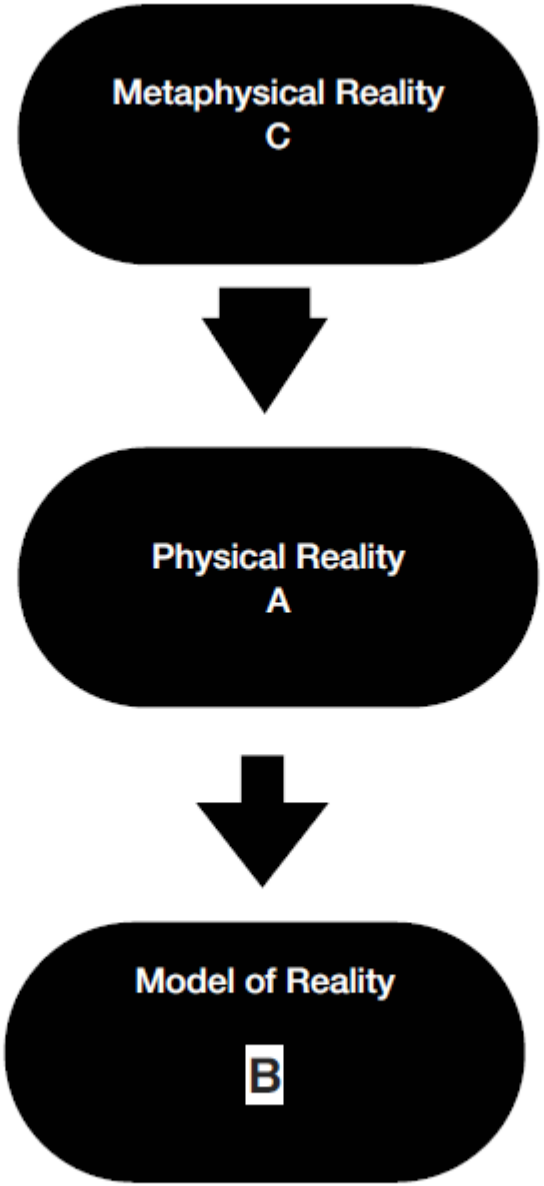
- Euclidean Space is not connected naturally: for the elements of a set are independent one of another
- Some arbitrary functions like the Peano conjectures can be applied to relate them at will but the Universe is inherently not arbitrarily connected but strictly related by a higher order relationship
- In order to represent this relationship formally it is necessary to rise above sets to Category Theory which in its pure form is a metaphysical language as opposed to the modelling language of Set Theory
- This problem with Set Theory is not only that the elements of a set are independent but also a set may not be a member of itself. Whereas a self reflexive category can be one of its own objects

Ordinary Numbers are only on the lowest rung of three Realities

- As a collection of categories the Universe may be viewed as a staircase of three rung processes interlocked by adjointness as a down from the meta-meta level These are continuously related downwards exponentially

Downward Exponential Process $A^C \rightarrow B^A$

This process $A^C \rightarrow B^A$ is under the control of the three level Heyting Logic



C × A ≤ B
=========
C ≤ A => B

This may be interpreted: **A** integral with its context **C** (such as the rest of the Universe) precedes **B.** That is mutually and conversely true with the requirement that **C** precedes the inference **A** implies **B**

Formally this is the inherent internal language of a pure Topos that has exponential objects but no ordinary numbers. That is empirically the same as the Universe. The pure Topos is to be distinguished from its earlier Grothendieck version whose logic relied on set theory

The Grothendieck in mainstream pure mathematics is therefore but a Boolean model of a pure topos

Ordinary Numbers

Ordinary numbers all belong in the lowest rung as axiomatic models

Whitehead insists that a system of axioms needs to be independent, consistent and be proved to exist. Peano needs around a dozen axioms to establish arithmetic

Zero is assumed as natural but never a successor number. Closure is also assumed under the equality relations (reflexive, symmetry, transitive) and the injective successor function by induction but there are issues over the mix between first and second order

Again the integers are make-believe only existing in the mind and not to be found in physics

Although claimed ‘real’ the ordinary numbers include the integers, the rationals and the irrationals all of which can be represented by a point on the straight line

There is still just that little problem that there are no such thing anywhere as straight lines other than in the mind

The = Sign

Innate connectivity in metaphysics imposes a comprehensive relationship between entities (represented by the arrow in Category Theory) covering all possible interpretations

One such effect is that every entity may be thought of as a first order model of every other entity and no model can be treated as wholly wrong

This belongs to Whitehead's 'every truth is a partial truth'

.

As empirically there are no two identical entities in the World the use of the equals sign '=' is a sleight of hand. Because all is process Whitehead claims that 'two times three equals six' should be read as 'two times three becomes six'. Clearly the left and right hand sides of the equation are

$$2 \times 3 = 6$$

are not the same. There is some assumed interpretation. This is one reason why Whitehead & Russell in their Principia had such difficulty in proving

$$1 + 1 = 2$$

The point being: how do we know that each of the two 'ones' have the same definition? Do any numbers exist?

Do any numbers exist?

Possibilities would still appear to be

$$0, 1, \sqrt{-1}(i), e, \pi, \infty$$

Ordinary Number Zero

- A special case is the number zero which is a mathematical or mythical creation as a cardinal number and again not to be found in the physical universe and therefore not prove to exist
- The arithmetical Zero has been invented many times throughout the World because of the need for a place-holder in any consecutive number system
- The algebraic zero on the other hand was invented by the early English polymath Thomas Harriott in the 16th century

Harriott on Quadratics

- He had some doubts about it because like all equations in physics there is always some issue whether the terms are pure numbers or have implied dimensions. To be safe Harriott would write a quadratic equation as

$$ax^2 + bx + c = 0,0,0$$

but realised that it was really an ordering for which he invented the sign although it does not help to relate the cardinals and the ordinals which have no natural correspondence

Problems with Nulls 1

- Zero has difficulties which are often swept under the carpet
  - The isomorphism with the null set is anomalous
- Compare the difficulties with null data values in databases
  - By the Closed World Assumption of relational databases any tuple not in the relation represents a false proposition
  - However this does not guarantee integrity of the tuples that are true

Problems with Nulls 2

- In particular the handling of nulls poses many semantic problems as while they are treated as data values, there are different interpretations such as
  - missing-but-applicable
  - missing-and-inapplicable
- Nulls therefore make a system undecidable
- Some more recent experimental versions of the relational model do not permit nulls, for example RAQUEL developed at Northumbria University

Problems with Nulls 3

The Council of the Royal Statistical Society has recently raised queries about the Null hypothesis in significance testing

Even more recently this year the Royal Astronomical Society has announced doubts about the zero point constant of the bolometric correction scale which is the basis for relative luminosity in astrophysics

This has far reaching effects on reported work for 80 years on luminosity with knock on doubts for understanding topics like the accelerating expansion of the Universe and general cosmology

Numbers Minus One and Infinity

Both the numbers minus one and infinity belong in the lowest rung of reality

The numeral -1 only provides a label to distinguish positive and negative numbers but its imaginary root ‘i’ goes further as an operator to express imaginary numbers in general but cannot be said to be truly a natural number

Likewise infinity ‘∞’ is a model in mathematics and does not exist anywhere in that form in physics

For 50 years, mathematicians have believed that the total number of real numbers is unknowable

However the corresponding concept of what Whitehead calls ‘unbounded’ is available in Category Theory as a free functor valued category and able to cope with the open closure of the Universe which is not possible with ordinary numbers

Number 1 is at the Top Level

- On the other hand the number ‘one’ is at the top of the three level ladder as a ‘really, really real number’
- How many true ‘ones’ are there in the Universe?
  - Because of non-severability there is no ‘one’ of anything
- There is therefore only one ‘one’ of the Universe, namely the Universe itself
- We use the self-reflective image of that ‘Universal one’ to carve out and designate ‘one of anything’ in the World

The top rung Really Really Real Numbers ‘1’, ‘e’ and ‘π’

- This use of ‘one’ is viable because the ‘really really real’ number ‘e’ relates objects recursively top-down
- It should not be surprising therefore that the ‘e’ is so fundamental to be needed in many physics energy equations where again it self-reflects down examples of mass/energy in the Universe

The top rung Really Really Real Numbers ‘1’, ‘e’ and ‘π’ (cont.)

- Of course the ‘really really real’ pièce de resistance is the number π which is profligate within nature to be found:
  - in so many equations for geometry with curvature such as infinite series & products, and integration in mathematics
  - in physics such from the simple pendulum to Einstein’s field equations, the cosmological constant, Heisenberg's uncertainty principle, electromagnetism, Kepler’s Laws of Planetary Motion, etc.

Ubiquity of π

Can we say we understand anything of fundamental science unless we can explain the ubiquity of π?

Here is a suggestion

It is not too well known that there is no equation for the perimeter of an ellipse. It cannot be accurately calculated or drawn in two or three dimensions. If you're clever you can calculate it approximately. The closest seems to have been evaluated by the prodigy Indian mathematician Ramanujan (1887- 1920)

The top rung Really Really Real Numbers ‘1’, ‘e’ and ‘π’

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Of course the ‘really really real’ pièce de resistance is the number π which is profligate within nature to be found: in so many equations for geometry with curvature such as infinite series & products, and integration in mathematics; in physics such from the simple pendulum to Einstein’s field equations, the cosmological constant, Heisenberg's uncertainty principle, electromagnetism, Kepler’s Laws of Planetary Motion, etc.

Examples of First Order Methods relying on Number

- First Order methods have been extremely successful for three centuries in dealing with local systems but become unnecessarily complicated on attempts to extend them at higher order like for global systems. This problem applies to swathes of mainstream science:
  - The limitations of that bottom level in science should be reviewed as it is blinkered.
- For example the common view is that the current Universe was initiated at the Bing Bang by quantum fluctuations — that seems to require quantum mechanics to have preceded the Universe!
- It has then continued up to today by some process of evolution with many Universes postulated, all posited firmly on the existence of Peano's number system.

Panel of possible types of Universes

Newton (Euclidean), Einstein (static gravity balance), Hubble (expanding), de Sitter (matterless acceleration), Friedman (matter in motion), Edington-Lemaître (infinite after ten + four accelerating billion years), Tolman (oscillating and ‘kinky’ inhomogeneous), D’Albe & Charlier (fractal), Kasner (Googleplex 10100), Dirac (gravity decaying), Einstein-Rosen (undulating), Milne (no expansion nor recession), Strauss (swish-cheese), Landau & Lifshitz (perturbed), Schrödinger (particle to wave), Gödel (recurring), Holmberg (table-top), Bondi & Lyttleton (electric), Gamow (hot), von Weizsäcker & al (turbulent), Bianchi & Taub (expanding spaces I-IX), Wilkinson & Partridge (smooth), Misner chaotic), Misner (mix in key), Thorne &al (magnetic), Brans & Dicke (gravitational weakening), Alfvén & Klein (antimatter), Hawking & Ellis (a singularity), No Success (cold & tepid), Politzer & al (high energy particles), Georgi & Glashow (Grand Unified Theory), Dirac (magnetic monopole), Guth (inflationary), NASA Satellites (chaotic inflationary), Current Mainstream (self-reproducing eternal inflationary), Post-modern (random, probable, anthropic, possible, home-made, naturally selected, fake, with nothing original, Boltzmann’s, wrap-around, quantum, self-creating, colliding, light dying, hyper-universes, best-buy, the preposterous, the puzzling).

Red Scientific Health Warnings 1

I. Quantum Mechanics elevated to Quantum Theory

linear differential Schrödinger equation to use as a model to form a misleading stable system instead of the higher order topos

II. Quantum Computing

the ‘quantum bit’ instead of the higher order ‘quantum monad’

III. Statistics and Probability

First order statistical models can be way out when applied to higher order phenomena. Compare grossly wrong predictions for the COVID-19 virus

IV. Newton & Gravity

Whitehead dismisses Newton’s First Law of Motion because of Euclidean limitations and the Second Law as a circular argument. Newton’s Third Law of Reaction needs to be recast with higher order Heyting logic

V. General & Special Relativity

Einstein’s powerful theories are neither right nor wrong but need to be recognised as only first order being derived from ordinary numbers

Red Scientific Health Warnings 2

VI. Chaos Theory

Mainstream treats first order chaos as disorganised ordinary numbers rather than a higher order phenomenon

VII. Pure Mathematics & Symbolic Logic

Mainstream is restricted mainly to local first order ordinary numbers whereas most problems today are at a higher order global level. Now it is being gradually recognised that much public finance is being wasted on useless ‘symbol-bashing’ of ordinary numbers. Leicester University has therefore recently closed down research in pure mathematics

VIII. Information Technology & AI

Both are currently limited by the restrictions of the Von Neumann machine architecture which is no more than a register that counts ordinary numbers up and down

IX. Climate change

Mainstream research on climate change is limited to first order ordinary numbers modelling the effect of human activity on the atmosphere which neglects the top level of natural processes of geophysics

X. Thermodynamics

The Laws of Thermodynamics are entirely axiomatic relying on ordinary numbers and are anomalous