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The system as emergent process in Topos Theory

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1 Overview

Emergence is a fundamental component of the current understanding of science but as a 'top-down' holistic mechanism it would be a contradiction in terms to call it foundational. Emergence describes the global devolution of the world everywhere adjoint to local evolution. Emergence is often as at this EMCSR meeting bracketed with the word 'design'. To grasp a rigorous understanding of these twin terms as part of science they need to be expressed as formal concepts. Systems Theory hardly justifies its existence as a theory if it cannot be expressed formally. Formal definitions are not always easy and particularly difficult when they lie at the cutting edge of science in *terra incognita*. Emergence is usually treated as a process and design as possessing an open structure. Words like 'process' , 'structure', and 'open' more fortunately do have a track record to draw on.

Emergence as allied to design belongs to open system theory. It is the process that comes out of 'openness'. To be scientific systems theory has to be underpinned by rigorous logic. Classical logic tends to be stuck in closed systems and does not avail much for open systems. Yet open systems comprise the vast majority of problems in current areas that system theory is today called to address. The logic of openness is by its very nature the logic of the third way. Such logic is very difficult to represent in classical logic because classical logic is Boolean and only operates two-way. We earlier drew attention to the difficulties even to define 'open systems'[1] let alone to understand them formally. A *cause célèbre* for emergent design is to explain the existence of life. The pioneer a way ahead of his time in the study of life systems was Robert Rosen who recommended a shift from set theory to category theory:

"the natural habitat for discussing . . . specific modelling relations". (Rosen 1991 p 153)

Rosen's informal diagram is reproduced in figure 1. Emergence is in effect the 'implication'[2] on the right of his diagram. We followed Rosen's prescription for category theory to express the logic of social systems[3] . It turns out that Rosen's informal diagram was an early attempt to represent the archetypal process of universal adjointness in a cartesian closed category as shown between any two categories in our corresponding formal diagram of figure 2. This shows the contravariant free and underlying functors between the left and right structures. Given any one of these fixes the other three uniquely when expressed this way. Therefore a particular right structure emerges by a particular choice of the free functor (F). This is both awesome and trivial in that every choice we make determines the next unique state of the world.

Figure 1 Rosen's Formal Modelling (Rosen 1991 fig 7F.1)

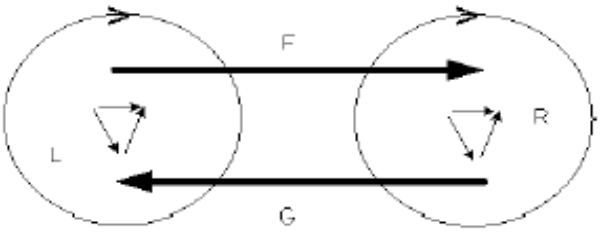

Figure 2: Adjointness between Left Category (L) and Right Category (R) for any left-adjoint free functor (F) and corresponding right adjoint contravariant functor (G)

(Heather & Rossiter 2006 p31)

However as a mathematical model is restricted by the limitations of set theory we have found it necessary to follow Alfred North Whitehead in his seminal work on Process & Reality and ascend up two levels from models to metaphysics[4]. This brings us to the highest possible level of category theory which is topos theory. The topos like the category is another of Aristotle's insights. From a conventional 'bottom-up' approach this is a terminal object but in the reality of quantum theory it is the starting point. The topos is the formal structure where every entity effects every other entity directly and indirectly through every other object, which describes the structure of the physical Universe. Grothendieke of the anarchic Bourbaki group of mathematicians in France was possibly the first to grasp the concept of the topos as a 'mathematical universe of universes'. However that is one level too short to reach the level of Whitehead's metaphysics. We need to go up to the category of category of categories (the level of the double power set in naive set theory) to reach the top level of closure, the topos as in figure 3. Here there are the three levels of category. The double headed arrow represents the pair of contravariant functors as in figure 2. Also such a pair exists between each level. Consequently the left to right structure of figure 2 represents the process of emergence.

[4] See, Heather & Rossiter (2014).

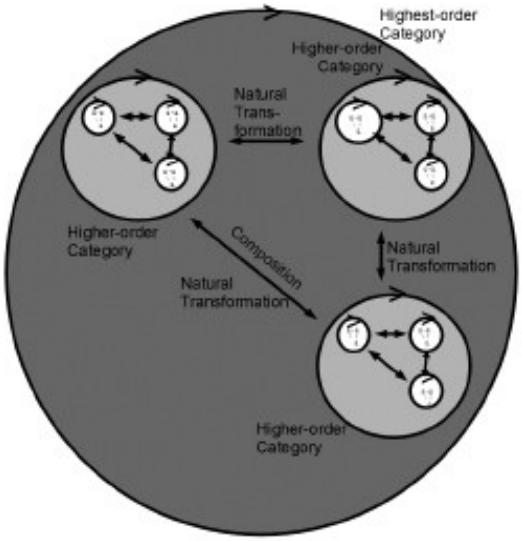

Figure 3 The Topos of Emergence

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About the Authors

Michael Heather and Nick Rossiter have collaborated on over 150 papers in the theory of information systems and related topics, concentrating mainly in the application of category theory.