Composed Categories and Monadic Adjunctions for Interoperable Structures

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Abstract. The use of first order predicate logic in many information systems may be justified through its completeness. However the work of Gödel shows that such systems are undecidable if they rely on formal systems of number and/or sets. For interoperability dyadic higher-order logic is required, which is neither complete nor decidable if based on sets. Category theory appears to be naturally suited to handling interoperability. However pure category theory is still axiomatic so is also neither complete nor decidable. Applied category theory based on cartesian closed categories for process is natural and appears to be both complete and decidable. Gödel's theorems therefore do not apply. Composed adjunctions appear particularly well-suited for modelling interoperability, with composition of distinct functors for mapping across a number of levels and of endofunctors for business process interoperability. The development of a tool based on categorial principles written in Haskell is proposed as a practical way forward.

1 Inherent Difficulty of Interoperability

Interoperability has proved to be a severe problem for information systems. Many avenues have been explored, as can seen by looking at the recent publication *Enterprise Interoperability* [2], including service-orientated interoperability, enterprise interoperability architecture, model-driven approaches to interoperability, methods, models, languages and tools for enterprise interoperability, semantics and ontology-based interoperability, interoperability of decision models, inter-organisational interoperability, interoperability of manufacturing enterprise application, business models interoperability and standards for interoperability. The plethora of approaches in itself suggests that none has had universal success outside of carefully controlled semi-automated local conditions. The root of the problem may lie in the mathematical basis for most information systems: set theory. This method has worked well in the past when the systems under examination were in general closed and the logic was that of a closed Boolean world. Today applied science has shifted down into things like nanotechnology and across into intangibles like information science and how humans behave, none of which is any longer within the easy ambit of classical physics. Society and medical science are concerned not just with interoperating parts of a system but with the relationship between parts and the system as a whole and with interoperability between systems through increasing globalisation, including between parts of one system and parts of another system. The major difference is that these systems have to be treated as open [19] and therefore not conveniently accessible by first order predicate logic.

1.1 The Results of Gödel

A highly desirable feature required for free and open systems theory is exactness. As we shall see below exactness can be formally defined but may be informally interpreted as 'certainty'. Probably the most rigorous path by which to approach certainty in logical foundations is through the work of Kurt Gödel that became a watershed in 20th century logic. There are two key concepts in Gödel's work which are components of 'certainty' and these are completeness and decidability. Gödel's 1929 doctoral thesis established that first-order predicate logic is complete [4], that is internally consistent.. This was followed the next year by his famous theorem of undecidability that applies to any system depending on axiom and number [3]. Gödel made three major contributions to logic that are very pertinent to the scientific method of the twentieth century. These are:

1. The system of first-order predicate (but not intuitionistic [7–9]) logic is complete [4, 5].
2. Any formal system of number and/or sets derived from axioms is undecidable [6].
3. The independence of the continuum hypothesis [1].

For such systems, cybernetic principles suggest a logic that permeates all three 'dimensions' of formal mathematics, empirical science and applied philosophy as enunciated by Husserl ([12] p.159) where just one or two on their own without all three together are insufficient. Husserl wrote around the turn of the twentieth century at the time when the logistical approach to mathematics was in vogue. Mathematics and logic had just been merged by Frege and the fine detail was being hammered out rigorously by Whitehead (1861-1947) and Russell (1872-1970) in their *Principia Mathematica* [22] in the belief that logic underpinned mathematics and there was really no more to mathematics than logic. It was at that same time around the 1900s, as Husserl [12] was sowing the seeds of post-modernism, that David Hilbert (1862-1943) was advancing the cause of the formalist approach that mathematics was wholly regulated by the manipulation of formulae irrespective of their meaning or interpretation. To this end he was presenting a formal Programme (with 23 research problems) of mechanical logico-mathematics for the modern world. Difficulties were there from the

[3] Gödel treated natural numbers and sets as equivalent because of the arithmetisation of sets [17]

outset like Russell's' paradox to raise doubts on the sufficiency of both Frege's axioms and Hilbert's programme but it was left to Gödel in the early 1930s [5–7] by his two theorems of undecidability to disprove the hope that any mechanistic axiomatic system of logico-mathematical principles (as Gödel referred to them) based on number or sets could ever be found. Husserl was also proved right because there were two of his 'dimensions' missing - the science and the philosophy.

1.2 Basis for Set Theory of Whitehead and Russell

We cannot apply Gödel's results properly without understanding logical foundations on which they are based. Gödel started with Russell and Whitehead's system [23]. The logico-mathematical basis for scientific reasoning is not clearly defined in mainstream work. If there is any consensus it is to be found within the tradition of Whitehead and Russell [23]. However, there is not even a standard version of these principles. For an analytical exposition of the principles of [23] it seems best to rely on the version given by Kurt Gödel. Because of the significance for all mathematical work and particularly because of applied mathematics for the rest of the twentieth century that rested on this foundation for reasoning itself, it is important to be aware of the nature of these principles consisting of formal axioms and rules of inference. Much if not all twentieth century mathematical models in science and engineering are postulated on them. They are nowhere uniquely defined but a typical list is given by Gödel himself as the starting point of his own work. He claims to rest on the propositions established by Whitehead and Russell denoted as *1 and *10 in their *Principia Mathematica*. Gödel reduced these to just eight axioms accompanied by four rules of inference ([4] p.67; [5] p.105).

The four rules of inference are:

1. The inferential schema: from the truth of $p \wedge p \longrightarrow q$, there may be inferred q.
2. The rule of substitution for propositional and predicate variables.
3. The inference for universal quantification of predicates.
4. Individual free or bound variables may be replaced subject to scoping.

Whitehead and Russell themselves however point out that there are many implied assumptions along the way such as the meaning of truth and falsehood and indeed the *Principia* is subject to tentative qualifications throughout the original work and even more equivocation and variance is introduced in the later second [23] and abbreviated edition [25].

A crucial principle in Whitehead and Russell's system of logic [23] is the Closed World Assumption with only the two Boolean possible outcomes. The upshot of these foundational axioms is that inference is defined only in terms of this Closed World Assumption. It means that negation, conjunction and disjunction are not independent. Although not mentioned by Gödel because he treats as given the assumptions of [23] nevertheless there are these fundamental definitions of true and false which are assumed by Whitehead and Russell. The first edition of the *Principia Mathematica* tells us we have to accept the concepts of truth, falsehood and the assumptions of the logical sum, logical product, complementarity and implication ([23] 1st ed. p.6). The later writings suggest that these four principles of deduction enumerated in [23] could be represented alternatively by five propositions ([20] p.149-150) although they do not explicitly correspond to those of Gödel. The second edition of [23] recognises that the four assumptions could be collapsed into one principle with the use of the Schaeffer stroke where $p \mid q$ is true if p is true or q is true or $p \wedge q$ is true, which is now further developed in the NAND operation. Whitehead & Russell [23] define as 'material implication' the concept $\neg p \vee q$. The Closed World Assumption or to give it its older Latin tag *tertium non datur* (there's no third way) is relied on by the *Principia* and those who depend on its inference schema to define inference itself that is the assertion of implication $p \longrightarrow q$ from $\neg p \vee q$. Scientific models therefore that draw scientific inferences are assuming the Closed World Assumption with all its ramifications.

2 Higher-order Logic for Interoperability

As we have already seen to justify the use of scientific models because they work only holds where they are close to a first order model (which will then satisfy first order predicate logic) and problems arising from Gödel's theorems of undecidability can be avoided. The scientific method of the last three centuries has actually achieved this by experimental verification. It is to be noted that this only holds locally and it is the completeness of first order predicate logic that gives such models their generality. For higher order and open systems experimental verification only holds locally without any guarantee of wider validity. Rather curiously the current prime promise to meet the requirements was developed by Alfred North Whitehead. This is process philosophy [24]. It appears that while Whitehead and Russell were collaborating on the *Principia* they had their doubts about fundamental entities [10]. This leads to a formal philosophy, but a metaphysics not a model, the common approach in theoretical computer science including artificial intelligence, which suffers from Gödel uncertainty.

Category theory provides a formal post-modern mathematics, bringing together algebra, geometry and topology. It is fully formal in its logico-mathematical representation so far as it is based on the empirical scientific principles for the particular category known as cartesian closed and embodies this philosophy of process as understood by Whitehead. Category theory achieves and goes beyond the post-modern mathematics sought by the Bourbaki French School of Mathematics [15].

3 Adjoint Functors for Scientific Basis

To escape the clutches of Gödel undecidability and to underpin our conceptual ideas, it is necessary to advance to cartesian closed categories beyond the category of sets to represent the relationship between different systems as adjoint functors. There are two particularly useful formal constructions for adjunctions in interoperability, both involving composition: the first that of distinct functors giving 2-cells, the second that of endofunctors giving monads. The former [18] represents the composition across a number of levels, for example composing data naming in turn with metadata and metameta data so that the adjoint relationship is represented across four levels of category, that is three levels of mapping, from data values to data abstractions such as aggregation and inheritance. The latter [11] represents the process or behaviour of a system, like in transactions, as an endofunctor in three cycles to give monads and comonads ([14] p.137-142). The two constructions are complementary: the first handling principally the data structures and their values and methods, the latter the behaviour of the data objects. It is interesting that three levels are involved in each construction: in limit constructions in category theory three levels are often used. The monadic structure has particular robustness with respect to Gödel's theorems. Monadic higher-order functions are complete and decidable unlike dyadic higher-order ones.

3.1 Composed Adjunctions: Distinct Functors

The application shown in Figure 1 involves the composition of adjunctions, that is an expression is derived in which two or more adjunctions are adjacent to each other. It is part of the power of category theory that adjunctions can be composed in the same way as other arrows.

The data functor (level pair) type change F maps target objects and arrows in the category $\mathbf{A}$ to image objects in the category $\mathbf{B}$ for each type of system. This mapping provides at the meta-meta level the data for each kind of system, that is to say how each abstraction is to be represented. We also label the functor pair $\bar{F}$ relating for each system the constructions in $\mathbf{B}$ with the names in a particular application in $\mathbf{C}$ and $\bar{\bar{F}}$ relating for each system the names in $\mathbf{C}$ with the values in a particular application in $\mathbf{D}$ The remaining functors G, $\bar{G}$ and $\bar{\bar{G}}$ are the duals of F, $\bar{F}$ and $\bar{\bar{F}}$ respectively. G for a given system relates the data modelling facilities provided by a system in $\mathbf{B}$ to the universal collection of abstractions defined in $\mathbf{A}$. $\bar{G}$ relates the schema definition in $\mathbf{C}$ to the constructs available in the system defined in $\mathbf{B}$. $\bar{\bar{G}}$ for a given $\mathbf{D}$ relates a data value type to its property name as defined in the schema $\mathbf{C}$. It will be noted that in Figure 1 all the mappings are two-way and that compositions naturally emerge.

$$\begin{tikzcd}
\mathbf{A} \arrow[r, shift left, "F"] & \mathbf{B} \arrow[l, shift left, "G"] \arrow[r, shift left, "\bar{F}"] & \mathbf{C} \arrow[l, shift left, "\bar{G}"] \arrow[r, shift left, "\bar{\bar{F}}"] & \mathbf{D} \arrow[l, shift left, "\bar{\bar{G}}"]
\end{tikzcd}$$

Fig. 1. Composition of Adjunctions

Then we may have six adjunctionss (if the conditions are satisfied):
$F \dashv G, \bar{F} \dashv \bar{G}, \bar{\bar{F}} \dashv \bar{\bar{G}}, \bar{F}F \dashv G\bar{G}, \bar{\bar{F}}\bar{F} \dashv \bar{G}\bar{\bar{G}}, \bar{\bar{F}}\bar{F}F \dashv G\bar{G}\bar{\bar{G}}$

These adjunctions give the following isomorphisms:
$\mathbf{D}(\bar{\bar{F}}\bar{F}Fa, d) \cong \mathbf{C}(\bar{F}Fa, \bar{\bar{G}}d) \cong \mathbf{B}(Fa, \bar{G}\bar{\bar{G}}d) \cong \mathbf{A}(a, G\bar{G}\bar{\bar{G}}d)$
where a is an object in $\mathbf{A}$ and d an object in $\mathbf{D}$. Each equivalent expression represents the collection of arrows from source to target so $\mathbf{D}(\bar{\bar{F}}\bar{F}Fa, d)$ represents the collection of arrows from $\bar{\bar{F}}\bar{F}Fa$ to d in category $\mathbf{D}$.

We can define these in more detail with their units and counits of adjunction as follows:

1. $< F, G, \eta_a, \epsilon_b >: \mathbf{A} \longrightarrow \mathbf{B}$
   η_a is the unit of adjunction $1_a \longrightarrow GFa$ and ϵ_b is the counit of adjunction $FGb \longrightarrow 1_b$
2. $< \bar{F}, \bar{G}, \bar{\eta}_b, \bar{\epsilon}_c >: \mathbf{B} \longrightarrow \mathbf{C}$
   $\bar{\eta}_b$ is the unit of adjunction $1_b \longrightarrow \bar{G}\bar{F}b$ and $\bar{\epsilon}_c$ is the counit of adjunction $\bar{F}\bar{G}c \longrightarrow 1_c$
3. $< \bar{\bar{F}}, \bar{\bar{G}}, \bar{\bar{\eta}}_c, \bar{\bar{\epsilon}}_d >: \mathbf{C} \longrightarrow \mathbf{D}$
   $\bar{\bar{\eta}}_c$ is the unit of adjunction $1_c \longrightarrow \bar{\bar{G}}\bar{\bar{F}}c$ and $\bar{\bar{\epsilon}}_d$ is the counit of adjunction $\bar{\bar{F}}\bar{\bar{G}}d \longrightarrow 1_d$
4. $< \bar{F}F, G\bar{G}, G\bar{\eta}_aF \bullet \eta_a, \bar{\epsilon}_c \bullet \bar{F}\epsilon_c\bar{G} >: \mathbf{A} \longrightarrow \mathbf{C}$
   $G\bar{\eta}_aF \bullet \eta_a$ is the unit of adjunction $1_a \longrightarrow G\bar{G}\bar{F}Fa$ and $\bar{\epsilon}_c \bullet \bar{F}\epsilon_c\bar{G}$ is the counit of adjunction $\bar{F}FG\bar{G}c \longrightarrow 1_c$
   The unit of adjunction is a composition of:
   $\eta_a : 1_a \longrightarrow GFa$ with $G\bar{\eta}_aF : GFa \longrightarrow G\bar{G}\bar{F}Fa$
   The counit of adjunction is a composition of:
   $\bar{F}\epsilon_c\bar{G} : \bar{F}FG\bar{G}c \longrightarrow \bar{F}\bar{G}c$ with $\bar{\epsilon}_c : \bar{F}\bar{G}c \longrightarrow 1_c$
   We have retained the symbol $\bullet$ indicating vertical composition as distinct from normal horizontal composition indicated by the symbol $\circ$ [13].
5. $< \bar{\bar{F}}\bar{F}, \bar{G}\bar{\bar{G}}, \bar{G}\bar{\bar{\eta}}_b\bar{F} \bullet \bar{\eta}_b, \bar{\bar{\epsilon}}_d \bullet \bar{\bar{F}}\bar{\epsilon}_d\bar{\bar{G}} >: \mathbf{B} \longrightarrow \mathbf{D}$
   $\bar{G}\bar{\bar{\eta}}_b\bar{F} \bullet \bar{\eta}_b$ is the unit of adjunction $1_b \longrightarrow \bar{G}\bar{\bar{G}}\bar{\bar{F}}\bar{F}B$ and $\bar{\bar{\epsilon}}_d \bullet \bar{\bar{F}}\bar{\epsilon}_d\bar{\bar{G}}$ is the counit of adjunction $\bar{\bar{F}}\bar{F}\bar{G}\bar{\bar{G}}d \longrightarrow 1_d$
   The unit of adjunction is a composition of:
   $\bar{\eta}_b : 1_b \longrightarrow \bar{G}\bar{F}b$ with $\bar{G}\bar{\bar{\eta}}_b\bar{F} : \bar{G}\bar{F}b \longrightarrow \bar{G}\bar{\bar{G}}\bar{\bar{F}}\bar{F}b$
   The counit of adjunction is a composition of:
   $\bar{\bar{F}}\bar{\epsilon}_d\bar{\bar{G}} : \bar{\bar{F}}\bar{F}\bar{G}\bar{\bar{G}}d \longrightarrow \bar{\bar{F}}\bar{\bar{G}}d$ with $\bar{\bar{\epsilon}}_d : \bar{\bar{F}}\bar{\bar{G}}d \longrightarrow 1_d$.
6. $< \bar{\bar{F}}\bar{F}F, G\bar{G}\bar{\bar{G}}, G\bar{G}\bar{\bar{\eta}}_a\bar{F}F \bullet G\bar{\eta}_aF \bullet \eta_a, \bar{\bar{\epsilon}}_d \bullet \bar{\bar{F}}\bar{\epsilon}_d\bar{\bar{G}} \bullet \bar{\bar{F}}\bar{F}\epsilon_d\bar{G}\bar{\bar{G}} >: \mathbf{A} \longrightarrow \mathbf{D}$
   The unit of adjunction is a composition of:
   $\eta_a : 1_a \longrightarrow GFa$ with $G\bar{\eta}_aF : GFa \longrightarrow G\bar{G}\bar{F}Fa$ with $G\bar{G}\bar{\bar{\eta}}_a\bar{F}F : G\bar{G}\bar{F}Fa \longrightarrow G\bar{G}\bar{\bar{G}}\bar{\bar{F}}\bar{F}Fa$
   The counit of adjunction is a composition of:
   $\bar{\bar{F}}\bar{F}\epsilon_d\bar{G}\bar{\bar{G}} : \bar{\bar{F}}\bar{F}FG\bar{G}\bar{\bar{G}}d \longrightarrow \bar{\bar{F}}\bar{F}\bar{G}\bar{\bar{G}}d$ with $\bar{\bar{F}}\bar{\epsilon}_d\bar{\bar{G}} : \bar{\bar{F}}\bar{F}\bar{G}\bar{\bar{G}}d \longrightarrow \bar{\bar{F}}\bar{\bar{G}}d$ with $\bar{\bar{\epsilon}}_d : \bar{\bar{F}}\bar{\bar{G}}d \longrightarrow 1_d$

The advantage in deriving these compositions is that we have the ability to represent the mappings in either abstract form to increase understanding or in detailed form to facilitate the development of a tool. The overall composition gives a simple representation for conceptual purposes; the individual mappings enable the transformations to be followed in detail at each stage and provide a route for implementation. The uniqueness of the components means that an adjunction can be resolved where there is a component missing.

If a further level **E** is added to Figure 1 with the adjoint $< \bar{\bar{\bar{F}}}\bar{\bar{F}}\bar{F}F \dashv G\bar{G}\bar{\bar{G}}\bar{\bar{\bar{G}}} >$, categorically the five levels are equivalent to the four levels above because composition is natural. The practical consequence is that a fifth level is equivalent to an alternative fourth level. So there is ultimate closure at a fourth (metameta) level.

3.2 Composed Adjunctions: Endofunctors

A monad is sometimes described as a triple, comprising an endofunctor say T, the unit of the monad η and the multiplication of the monad $\mu : T^2 \longrightarrow T$:

$$\text{Monad} =< T, \eta, \mu >$$

A pair of adjoint functors is an endofunctor: in this case the source category of F, **L**, is also the target category of G. So for the endofunctor T as the pair of adjoint functors GF, $F : \mathbf{L} \longrightarrow \mathbf{R}$ and $G : \mathbf{R} \longrightarrow \mathbf{L}$:

$$\text{Monad} =< GF, \mathbf{1_L} \longrightarrow GF, GFGF \longrightarrow GF >$$

where $\mathbf{1_L} \longrightarrow GF$ is the unit of the monad and $GFGF \longrightarrow GF$ is the multiplication.

The monad gives the left-hand perspective. There is also a dual comonad which gives the right-hand perspective, necessary to represent the changes to notional time. A comonad is a triple, comprising an endofunctor say S, the counit of the comonad ϵ and the comultiplication of the comonad $\delta : S \longrightarrow S^2$:

$$\text{Comonad} =< S, \epsilon, \delta >$$

A pair of adjoint functors is an endofunctor: in this case the source category of G, **R**, is also the target category of F. So for the endofunctor S as the pair of adjoint functors FG, $G : \mathbf{R} \longrightarrow \mathbf{L}$ and $F : \mathbf{L} \longrightarrow \mathbf{R}$:

$$\text{Comonad} =< FG, FG \longrightarrow \mathbf{1_R}, FG \longrightarrow FGFG >$$

where $FG \longrightarrow \mathbf{1_R}$ is the counit of the comonad and $FG \longrightarrow FGFG$ the comultiplication.

The diagram in Figure 2 assists with interoperability as follows. There is a unique solution, ensuring reproducibility, through the adjointness $F \dashv G$. The displacements in the left category η and the right category ϵ are given by the monad and comonad respectively. If there is no displacement in the left- or right-categories, that is η maps onto $\perp$ and $\top$ maps onto ϵ, then the relationship is

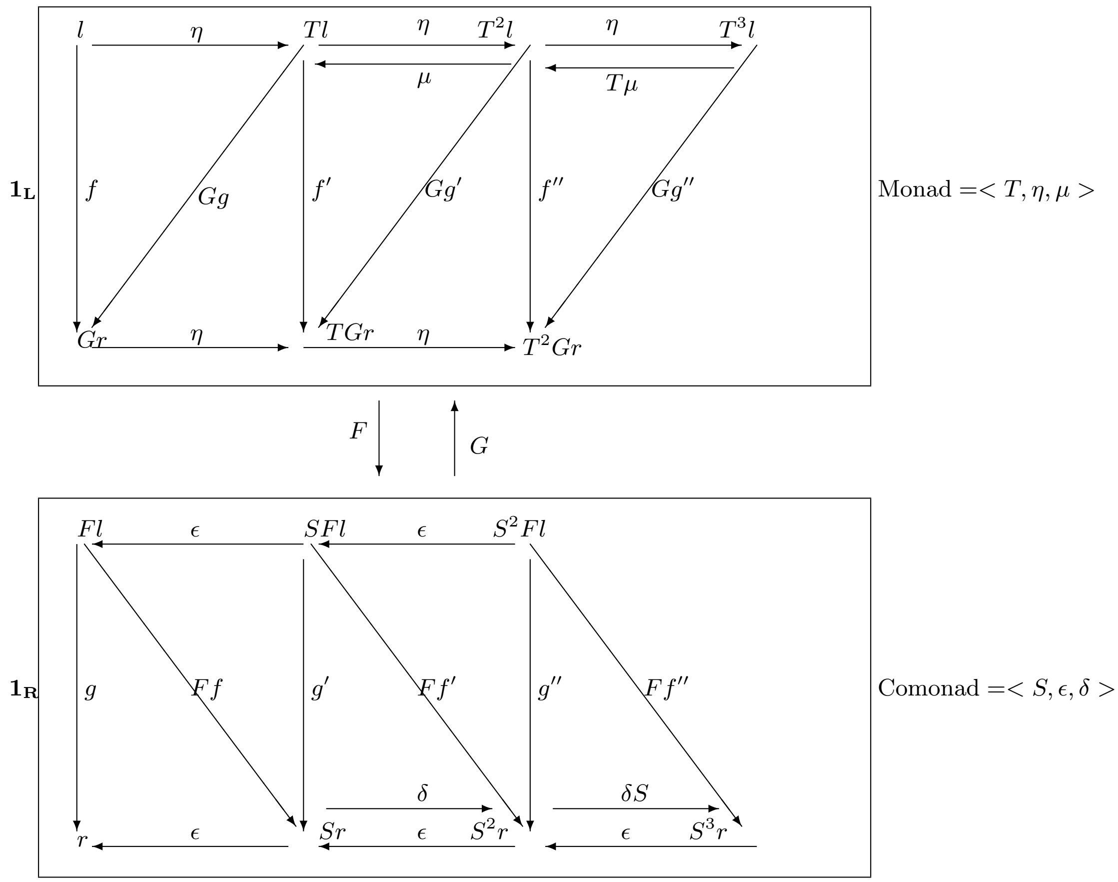

Fig. 2. After three cycles $GFGFGF$ from left-hand category and three cycles $FGFGFG$ from right-hand category: η and δ map onto other than $\perp$, $\top$ maps onto other than ϵ and μ

the special case of equivalence between F and G and the two categories are isomorphic. Determinism is measured through the arrow $\mu : T^2 \longrightarrow T$ (looking back). Closure is achieved through the third cycle with $T\mu$ ($GFG\epsilon F$) comparing the second and third cycles from the viewpoint of the third cycle (looking back) and δS ($FGF\eta G$) comparing the second and third cycles from the viewpoint of the second cycle (looking forward). If $GFG\epsilon F$ maps onto id_{T^3} and id_{S^2} maps onto $FGF\eta G$ then the relationship is the special case of equivalence between F and G and the left- and right-categories are synchronised. Anticipation is measured through the arrow $\delta : S \longrightarrow S^2$ (looking forward). This arrow as a free functor is non-deterministic.

4 Practical Significance

In terms of the various levels of interoperability recognised by the EU and their working parties, composed adjunctions with distinct functors deliver semantic interoperability by relating data values to metameta data. Composed adjunctions for endofunctors provide a route through to the more challenging business interoperability by delivering a description of process. Dynamic composition of services has been proposed [16] as a way forward for interoperability and this would benefit from a categorial approach.

To apply categorical techniques it should not be necessary for users to have an understanding of category theory. Rather the goal should be to develop tools, based on category theory, that assist users in providing interoperability between systems. A tool based on sound mathematical principles is more likely to provide the basis for a standard [11]. As Egyedi noted ([3] p.562):

> [We] explored why standard-compliant products often do not interoperate and what solutions are possible. Although the problem usually lies in the way standards are implemented, most of the underlying factors are located earlier in the standardization chain, namely either in a weakness in the standards ideas, the standards process or the standard specification.

Such tools could be written in any language but in practice some languages are more suitable than others. Simple functional languages are hardly sufficient for the multi-level category theory but functional languages with multi-level capabilities such as the ability to represent higher order logic as a basic construction look much more promising. In this respect Haskell would be a strong contender, particularly as it has the monad construction already available as a first-class structure [21].

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