Information Systems and the Physical World

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Outline

- Formal representations of real world
  - Based on information systems
  - Look at underlying assumptions
    - How questionable are they?
  - Consider maths in terms of underlying physics
    - Increases our confidence
- Review formal structures
  - Locally Cartesian closed category (LCCC)
    - Underlying data structures
  - Cartesian monad
    - Unification of categorial structures and manipulation

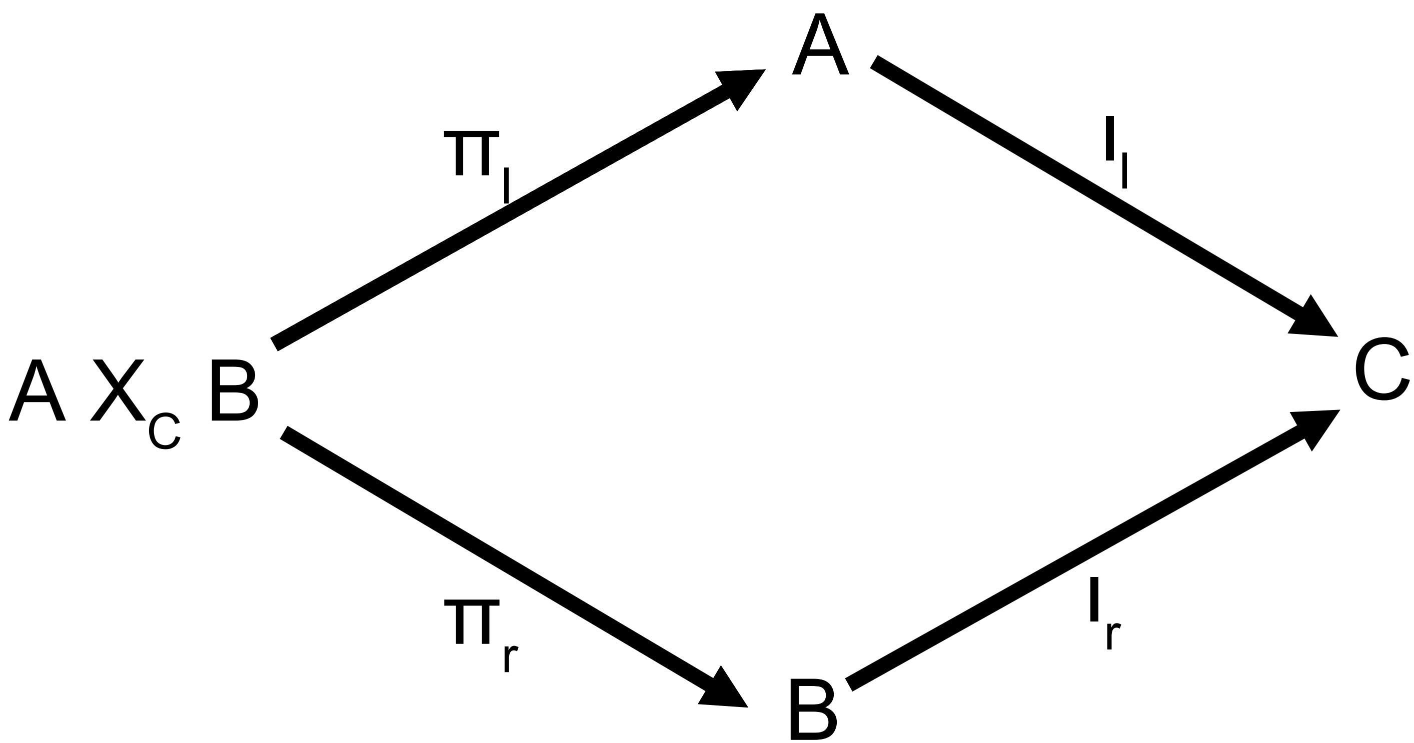
Formal Representation

- Based very much on
  - Cartesian closed category (CCC)
    - Connectivity (exponential)
    - Product (prerequisite for relationships)
    - Initial object (unique starting point)
    - Terminal object (unique finishing point)
  - Fits in with philosophy
    - Everything is connected
    - Everything is related
    - Everything is limited

LCCC

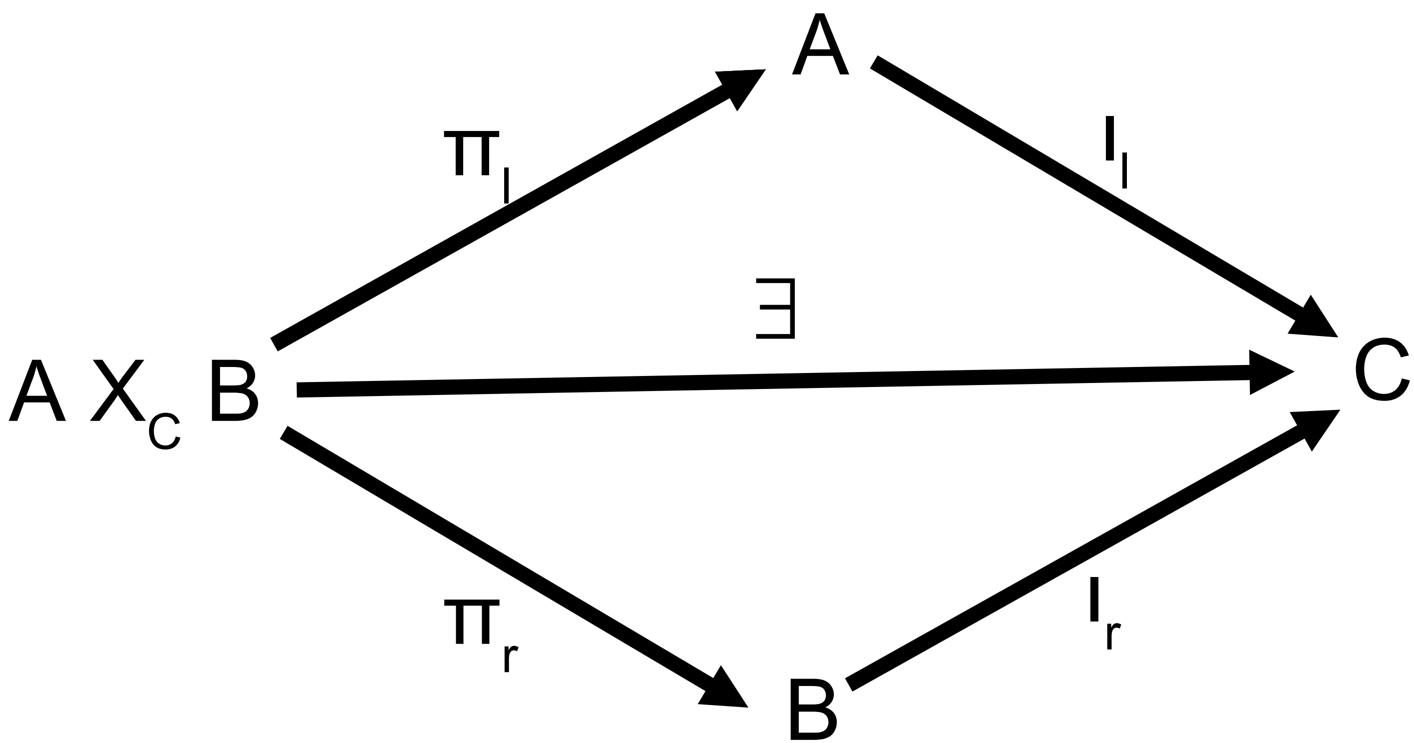
- In practice we use a variant of Cartesian closed categories
  - Locally Cartesian closed category (LCCC)
    - Product is replaced by a relationship
  - Product is all possible pairs
    - e.g. account number X borrower name (A X B)
  - Relationship is those pairs that satisfy a particular context
    - e.g. account number X borrower name in the context of cash owed ($A\ X_C\ B$)
  - In category theory this is a pullback (with adjointness properties)

Pullback



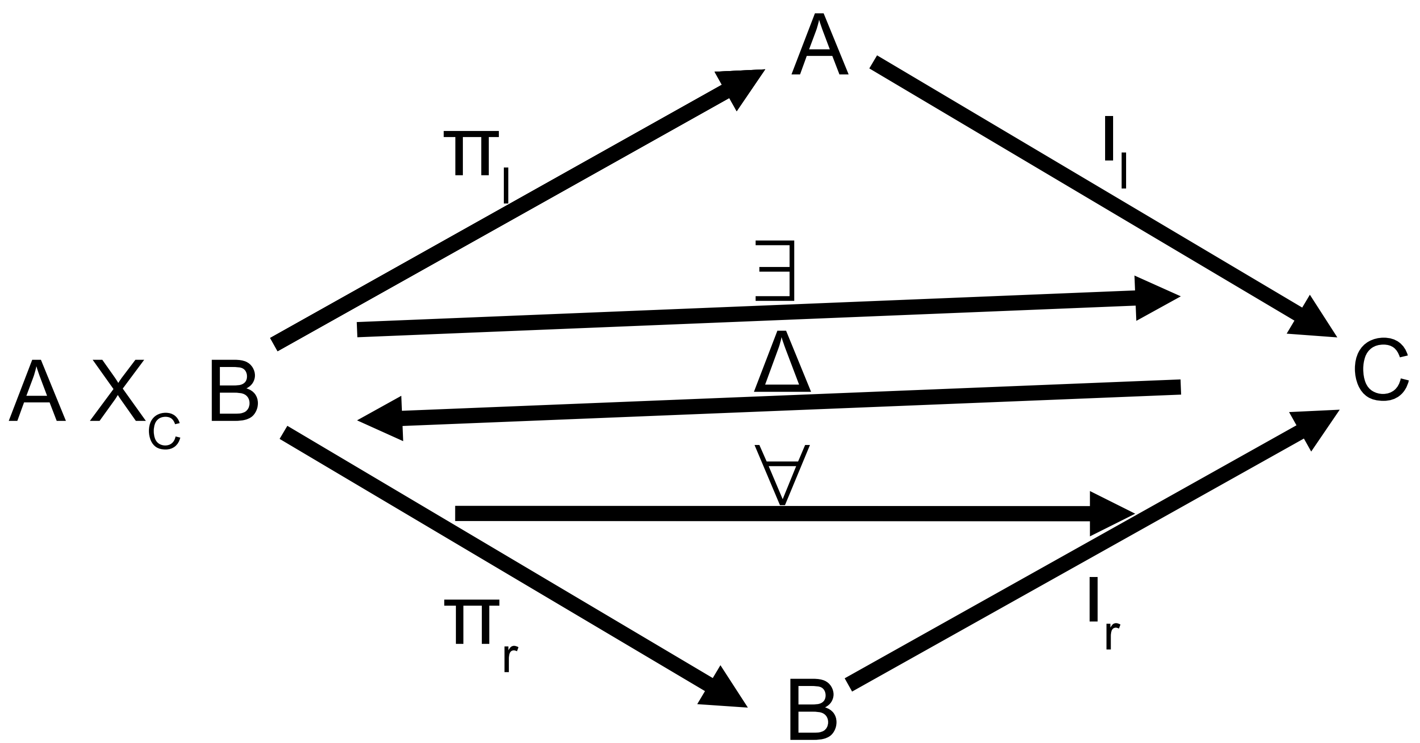
C is A+B+C

Pullback



$\exists$ is an equaliser: $\exists = \iota_l \circ \pi_l = \iota_r \circ \pi_r$

Pullback



Adjointness requirements ∃ ┤ Δ and Δ ┤ ∀

Working Assumption

- The Pullback has underpinned much of our work on information systems
- But is this justified?
- Information systems are open ended.
- We cannot prove all our instances of data are pullbacks.
- But we can try to relate pullbacks to accepted practice in software engineering.

Software Engineering Principles

- Information system data design
  - Normalisation Commonly to 3NF (third normal form)
- Process design
  - High coherence
  - Low coupling
  - Transaction
- How do these concepts relate to LCCC?
- LCCC have been popular in theoretical computing science
  - But little attempt to handle design issues

Normalisation Outline

- A relation comprises a collection of attributes
  - e.g. delivered (customer_id, customer_name, customer_address, item_code, driver_id, driver_name)
- Decide on those that provide uniqueness and make these the key
  - customer_id, item_code
- The others become non-key
  - customer _name, customer_address, driver_id, driver_name
- Requires knowledge of how things are done physically

Normalisation Stages

- Then check validity against 3 forms of increasing severity:
  - 1NF: for relation R each non-key attribute is functionally dependent on the key
  - 2NF: R is in 1NF and each non-key attribute is fully functionally dependent on the key (not dependent on any component of key)
  - 3NF: R is in 2NF and no non-key attribute is functionally dependent on another non-key attribute
- Maths in set theory is convoluted – students find it challenging. e.g. Ullman, J D, Principles of Database and Knowledge-base Systems (1988).
- Some category theory work has tried to directly represent set approach in categories – categorification
  e.g. Johnson, M, & Rosebrugh, R, Sketch Data Models, Relational Scheme and Data Specifications, Electronic Notes in Theoretical Computer Science **61** 51-63 (2002).

1NF

- A relation is in 1NF if there is a functional dependency from the key to each non-key attribute.
- So expectation is:

customer_id, item_code → customer _name

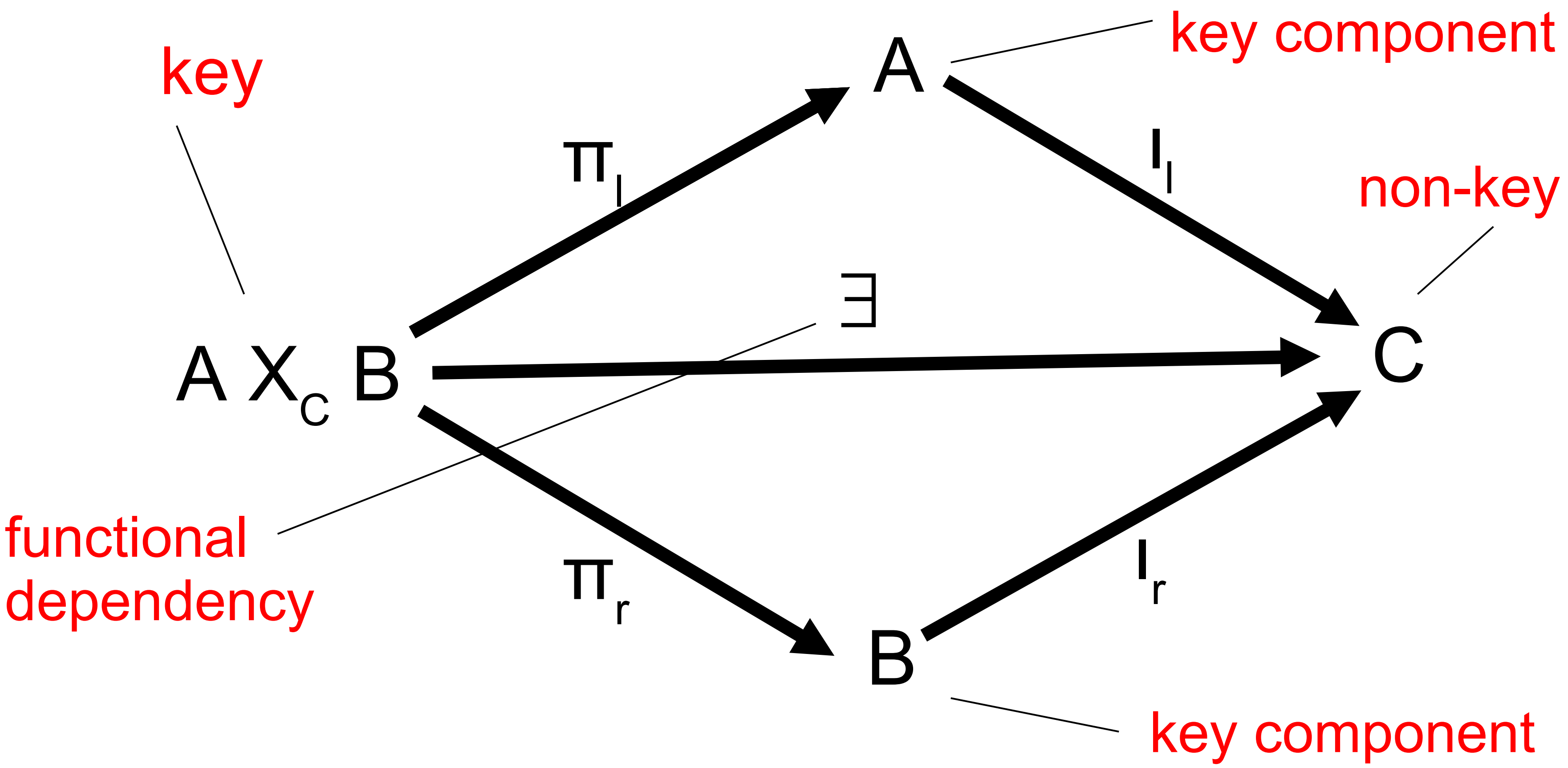
customer_address

driver_id

driver_name

If add something unrelated such as football_club then not in 1NF: need everything to be connected

LCCC view of 1NF - Pullback

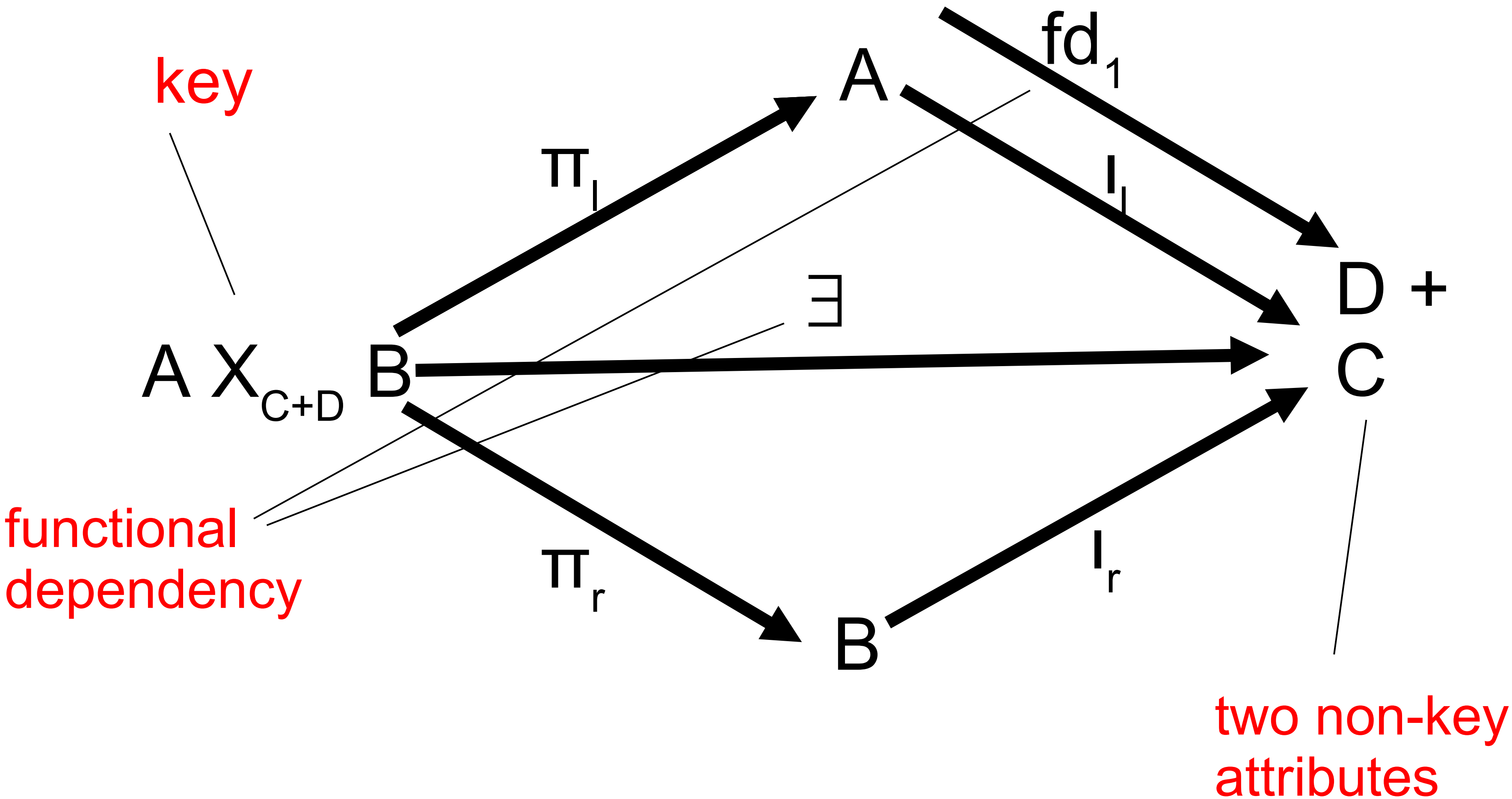


All attributes must be related;
adding stand-alone attributes means it's not even CCC

1NF is insufficient

- Everything is connected
- But may not be connected optimally
    - May be other arrows
        - From key component to non-key as a functional dependency
        - From non-key to non-key as a functional dependency
- Tests for these arrows are done in 2NF and 3NF respectively
- Potential presence of these unwanted arrows means that the diagram is not yet a LCCC

Introducing arrow to invalidate 2NF



$fd_1 : A \to D$; $I_l : A \to A + B + C + D$;

adding fd_1 means that component of key determines non-key

Example of failing 2NF relation

Functional dependencies below are from component of key to non-key

customer_id → customer_name
customer_id → customer_address

Vast duplication of customer data each time something is delivered

Not a Valid Category, let alone LCCC

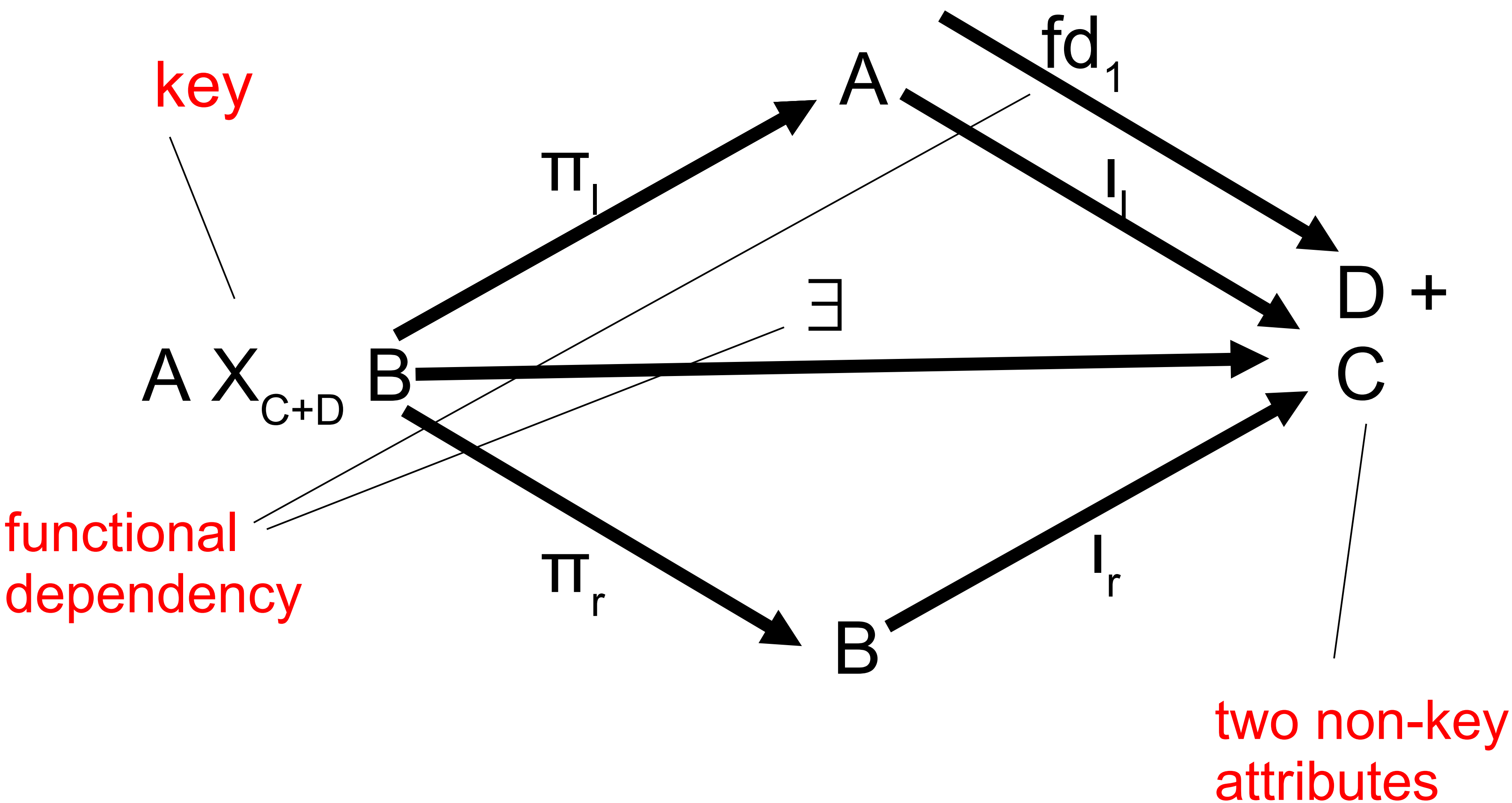
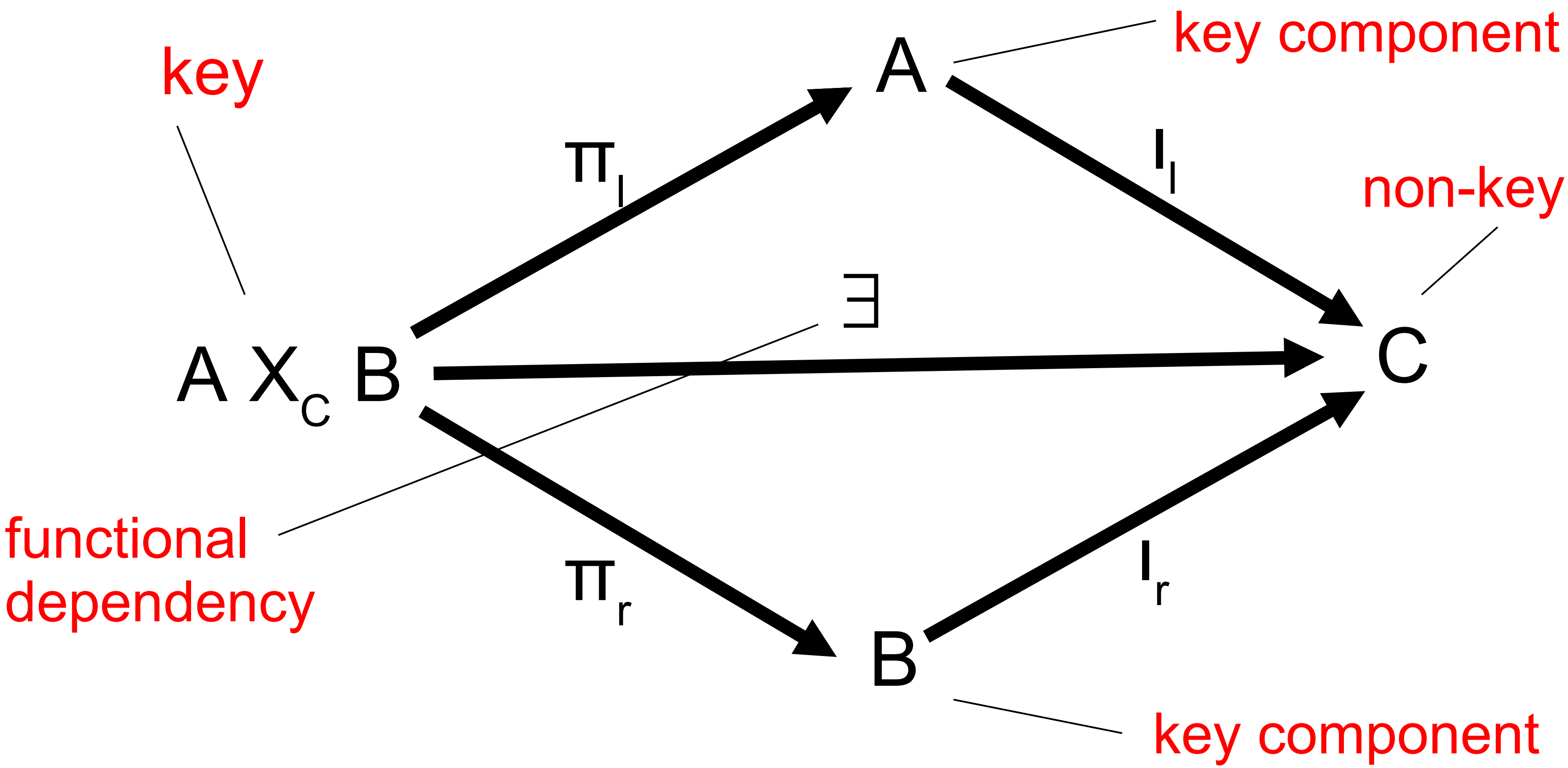


Diagram does not commute. D+C obtained by following top path does not equal that obtained by following bottom path.

Solution

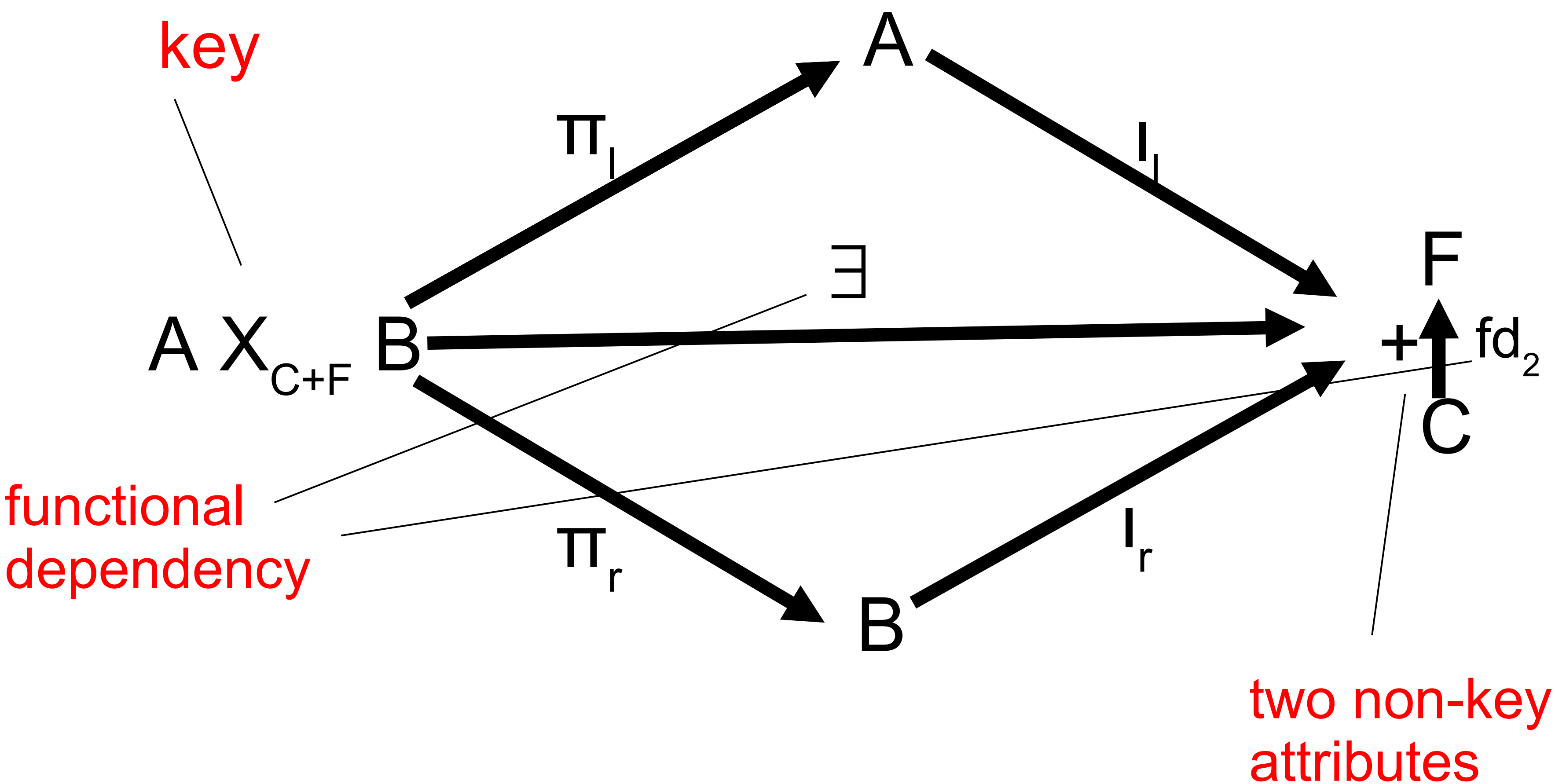
- Take A → D arrow out of pullback diagram
- Insert A → D dependency within category A, giving A more internal structure
- A (or B) can be an object or a pullback category with identity functor for reference purposes
- Alternative: possibly paste an additional pullback onto previous structure.

LCCC view of 2NF - Pullback



Category A contains dependency $fd_1 : A \rightarrow D$

Introducing arrow to invalidate 3NF



$fd_2 : C \rightarrow F$;

adding fd_2 means that one non-key determines another non-key

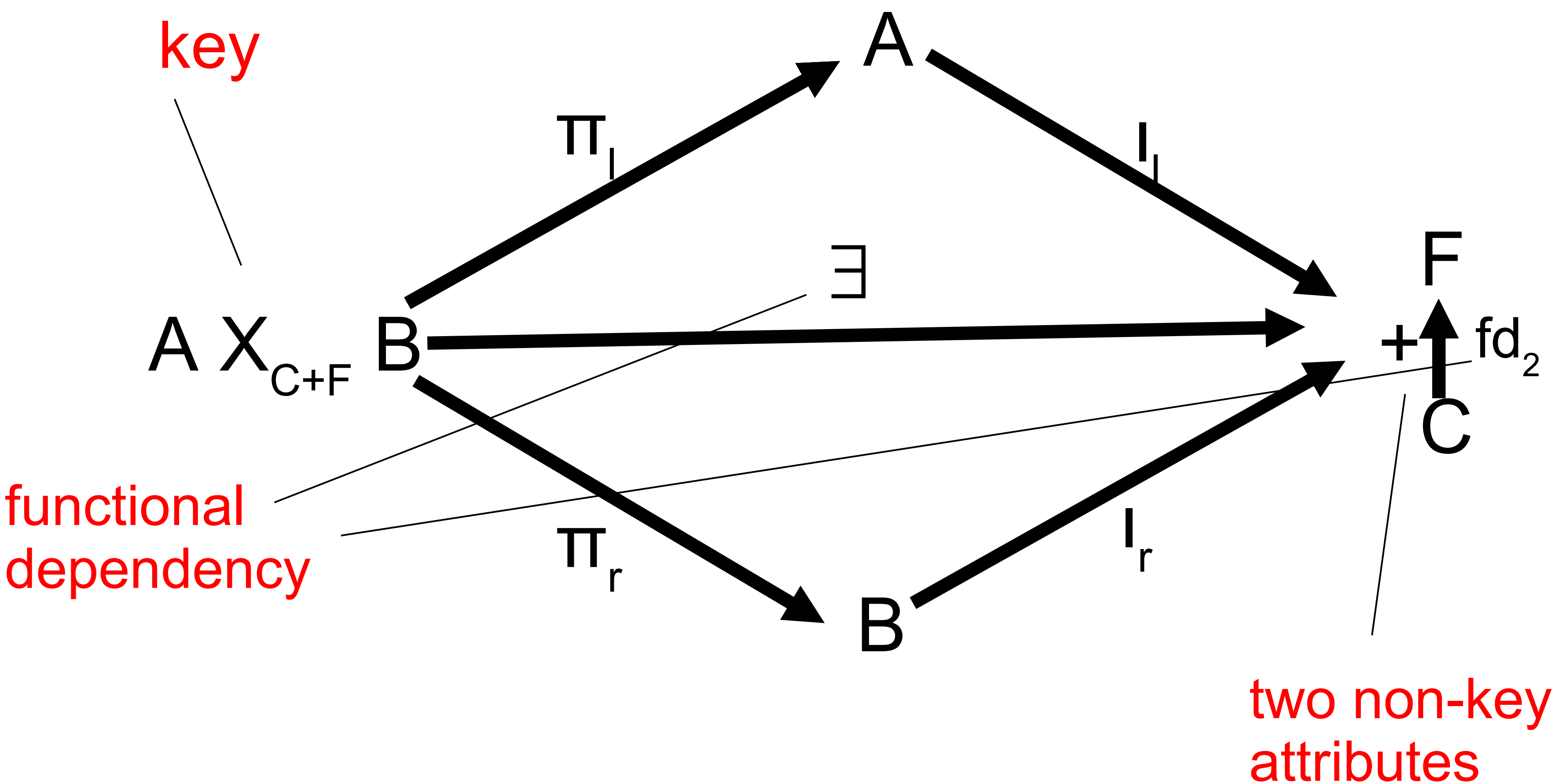
Example of failing 3NF relation

Functional dependencies below are from non-key to non-key

driver_id → driver_name

Vast duplication of driver data each time something is delivered

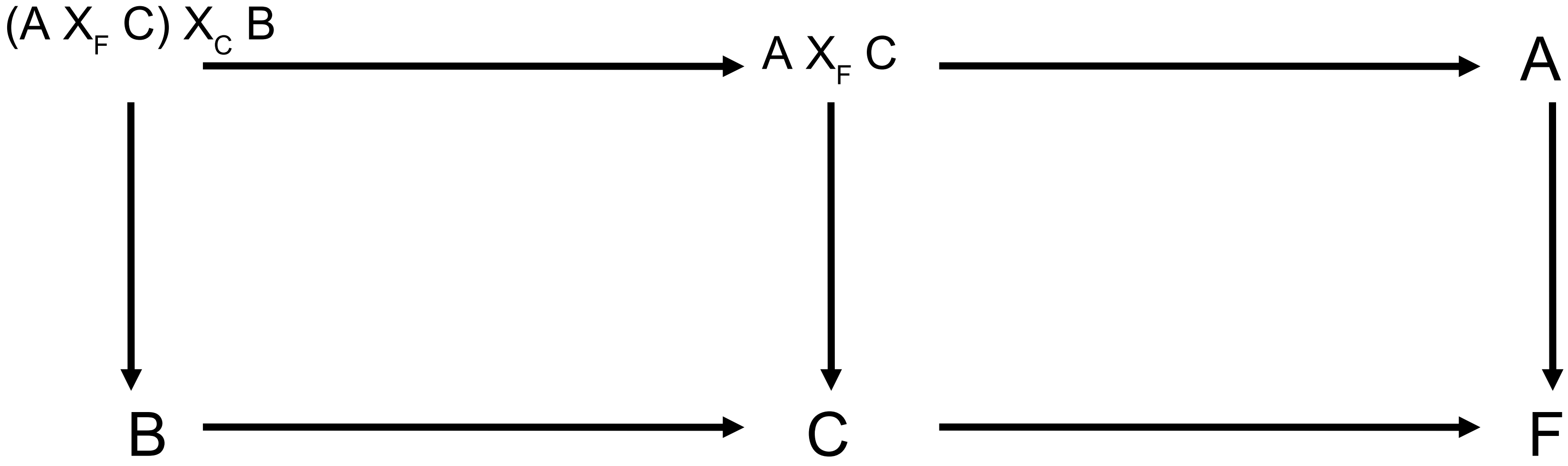
Not a Valid LCCC (Pullback)



Terminal object should be A+B+C+F (typed as a disjoint sum);
May not even be a category (depends on how constructed)

Solution

- Take C → F arrow out of pullback diagram
- Develop new pullback to represent relationship between C and F
- Paste new pullback onto existing structure.

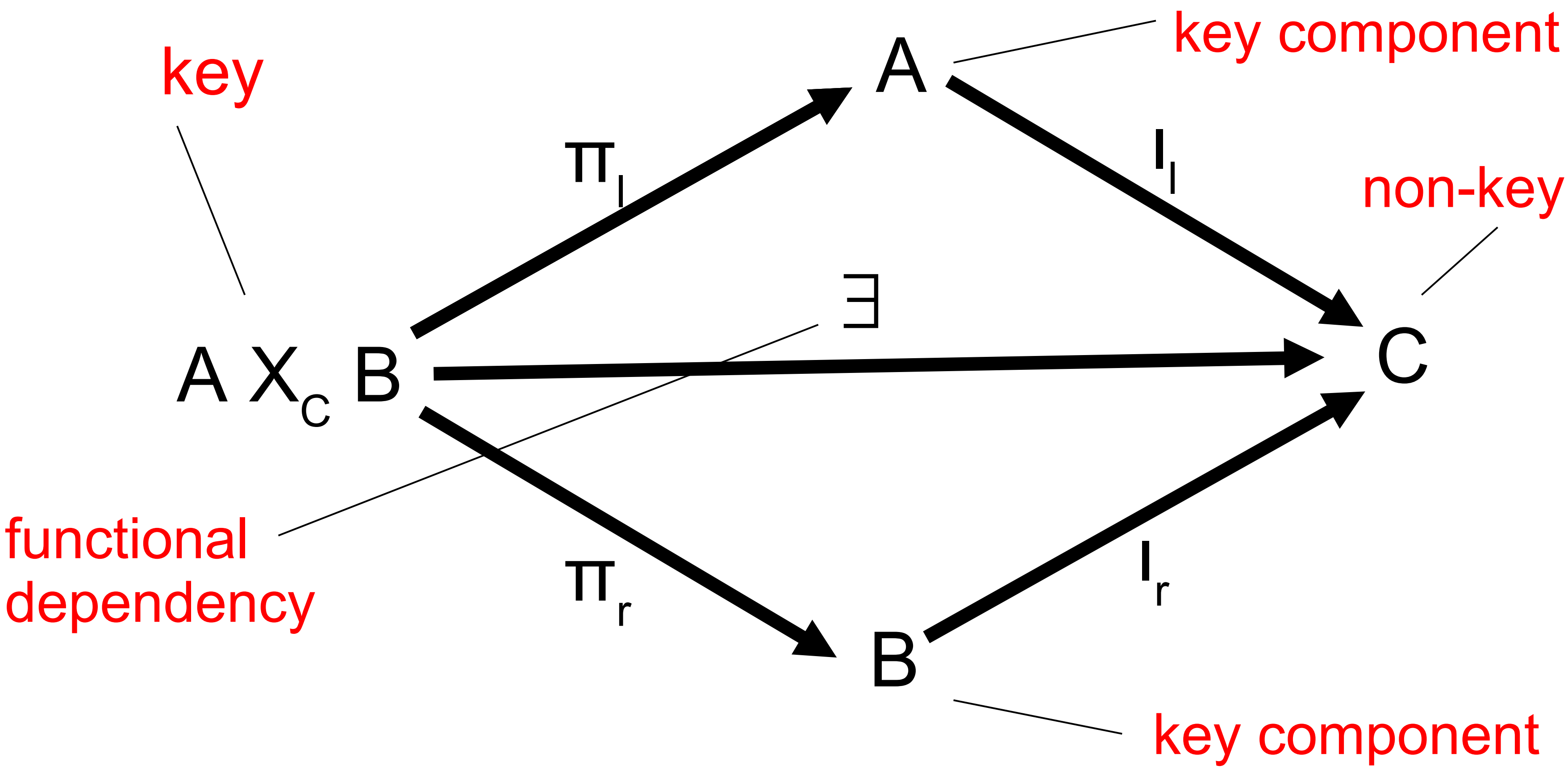


3NF and LCCC

- 3NF (non-stepping stone via 1NF and 2NF)
    - A relation is in 3NF if each non-key attribute is dependent on the key, the whole key and nothing but the key

- LCCC
    - A relation is in 3NF if a valid pullback can be constructed from its functional dependencies

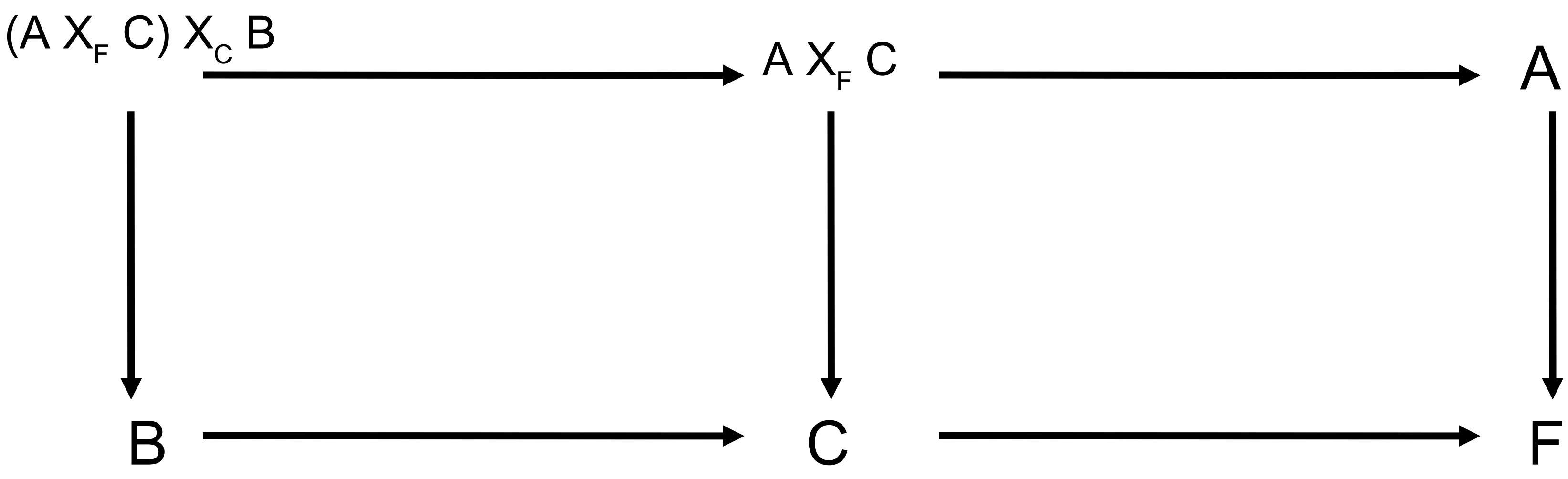
LCCC view of 3NF – Single Pullback Diagram



No other arrows permitted

LCCC view of 3NF – Pasted Pullback Diagram

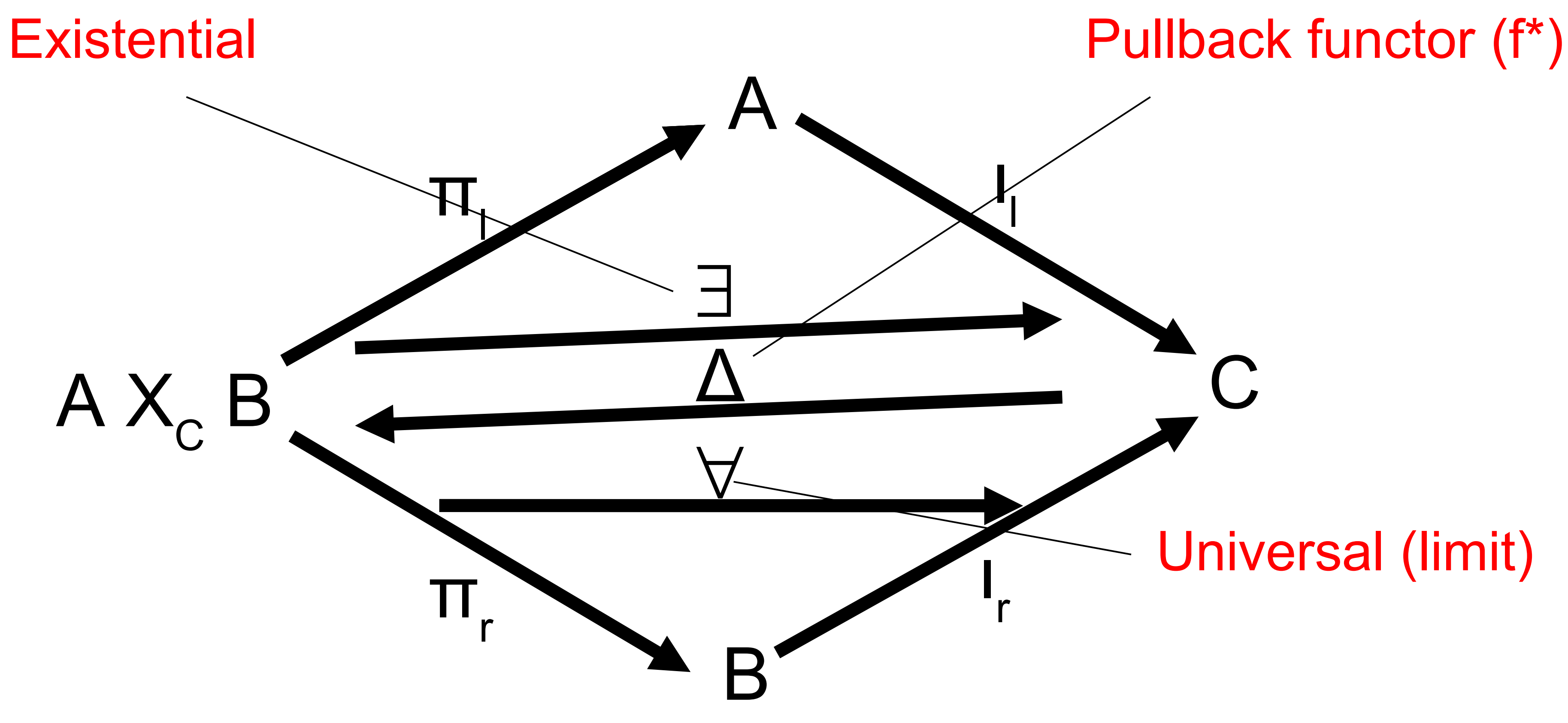
Complex pullback diagrams can be pasted together as below
Format of squares as below must be respected
No other arrows allowed



Higher Normal Forms

- In database theory go up to Boyce-Codd, 4NF and 5NF.
  - But 3NF is industry standard
- 5NF is Project-Join Normal Form
  - Define relations so that projection of attributes followed by joining together again returns starting point
- Already provided by LCCC in the adjointness between the X side and the + side.

LCCC for 5NF



Adjointness ∃ ┤ Δ and Δ ┤ ∀ between functors mapping between X and + (project-join)

Interesting Points

- So assumption that LCCC is a satisfactory basis for information system representation is justified by its close correspondence to data normalisation at industry standard (and beyond)
- Data normalisation has a sounder basis in LCCC than in set theory
  - Conceptual bases conform naturally
    - Arrows naturally handled with categories
  - All normal forms up to 5NF are handled in a single diagram
  - LCCC provide a springboard for further data semantics

Class Model Constraints as LCCC Types

Arrow	Epic (surjective)	Membership class	Monic (injective)	Cardinality
π_l	Y	A mandatory		
	N	A optional		
π_l^* (* is inverse)			Y	Each A onto 1 relation instances
			N	Each A onto N relation instances
π_r	Y	B mandatory		
	N	B optional		
π_r^*			Y	Each B onto 1 relation instances
			N	Each B onto N relation instances

Software Engineering – Process

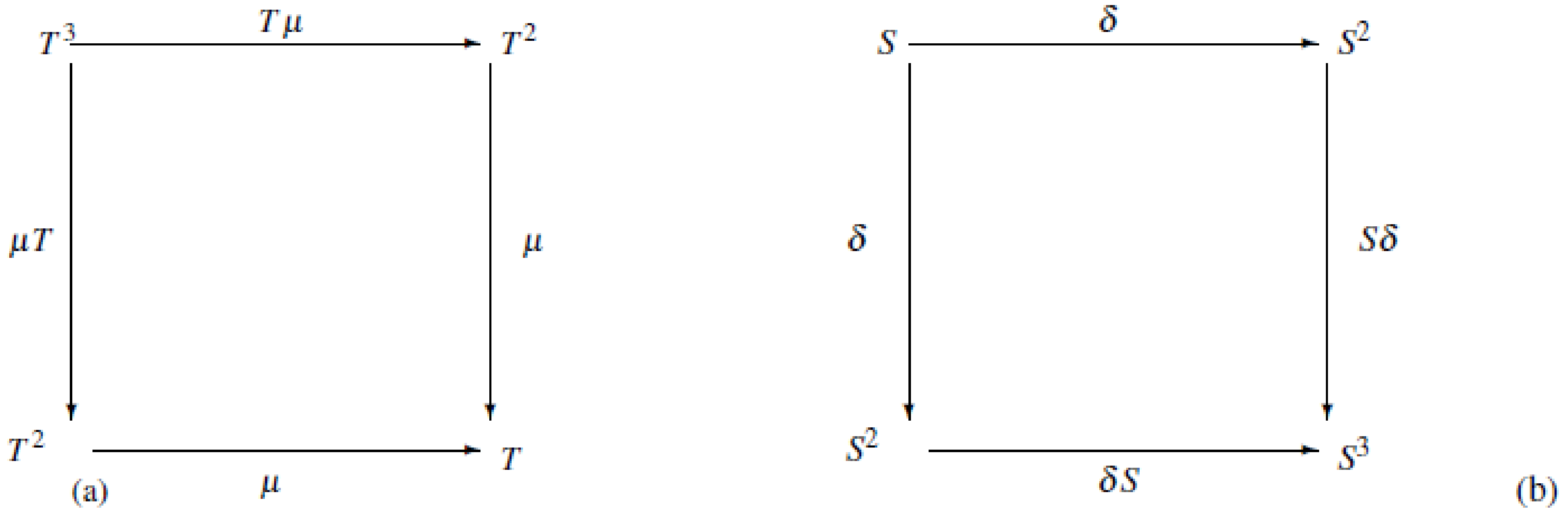
- Principles include
  - High cohesion
    - Everything is connected
      - Cartesian closed category
  - Low coupling
    - Entrance is always through official interface
      - Initial object in Cartesian closed category
    - Exit is always through official closure point
      - Terminal object in Cartesian closed category
- So less formal than with structures but some properties of CCC

Software Engineering – Transaction

- Transaction is standard way of defining a process
    - Principles of ACID
        - Atomicity, Consistency, Isolation, Durability
    - Logical technique for controlling the physical world e.g. banking transaction
- Requires three cycles of adjointness between initial and target state
    - First two for atomicity, consistency and isolation
    - Third for durability
- Process as a World Transaction, same authors as this paper, 36pp ANPA(2006).

Transaction ~ Monad/Comonad

- In category theory transaction is effectively represented by a monad/comonad pairing



a) Associative law for monad <T,η,μ >; b) Associative law for comonad <S,ε,δ>

Monad/Comonad

- Functionality
  - Monad (looking back over 3 cycles)
    - $\mu : T^2 \rightarrow T$ (multiplication)
  - Comonad (looking forward over 3 cycles)
    - $\delta : S \rightarrow S^2$ (comultiplication)
- Objects of monad/comonad
  - Adjoint pair of functors between initial and target state
  - Initial and target state are LCCC (pullbacks)

Cartesian Monad

- If underlying categories are pullbacks

AND T preserves pullbacks

AND μ and η are Cartesian

Then the monad is a Cartesian monad

- That is, the underlying structures and the manipulation language are unified into a single categorial concept
    - The relational model (with sets) elevated to a categorial representation much closer to the physical world

Summary

- LCCC are indeed justified as the choice of category for representing information systems
  - Data structures as pullback
    - Data normalisation
      - to 3NF industry standard and beyond to 5NF
    - Typing of class model constraints
      - Membership class
      - Cardinality
  - Manipulation as Cartesian monad/comonad on pullback
    - Transaction
      - Unification with data structures

Advantages of LCCC over Sets

- 3NF is achieved directly through the pullback construction
    - Not through an optional design process of normalisation, unenforced in relational database systems
- Class model constraints are typed in the arrows of the pullback
    - Not labelled as in the Entity-Relationship model
- Manipulation by transactions is unified
    - Not with impedance mismatch of relational systems