

# Don't Confuse Left and Right Exactness!

## A Note correcting the Distinction in the Proceedings of ANPA 27

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### Abstract

A note correcting and enlarging on the distinction between the parities of exactness and adjointness in our paper *Process as a World Transaction* in the proceedings of ANPA 27.

## 1 Background

At ANPA 27 we went through the details for the withdrawal of cash from an Automated Teller Machine as a database transaction that could not be perfectly achieved with present computers based on the von Neumann architecture. Category theory is able to show the difference between physical process and its formal representation using classical mathematical models. However in the published version of the written paper [3] the distinction between left/right adjointness and left/right exactness is not properly distinguished in our Concluding Summary of the paper in section 10 at page 153-154. The corrected version of this *Concluding Summary* is reproduced below. We have also taken the opportunity to replace the Figure 20 with a more recent improved version that makes patent the covariant/contravariant distinction between the two triangles in the left and right categories.

The diagram in Figure 20 employs the possible uses of left and right to be found in categorial descriptions. Confusion can arise from thinking in terms of labels left and right. The concepts themselves are always quite distinct. In our ANPA 27 paper the final concluding summary became too compressed. The correct version of section 10 is shown below, with appropriate changes made to item 1.

## 10. Concluding Summary

Because adjointness is everywhere and the discussion has been wide-ranging we should perhaps conclude with a summary of the main theme of this paper of transaction in information systems as process. The main steps are:

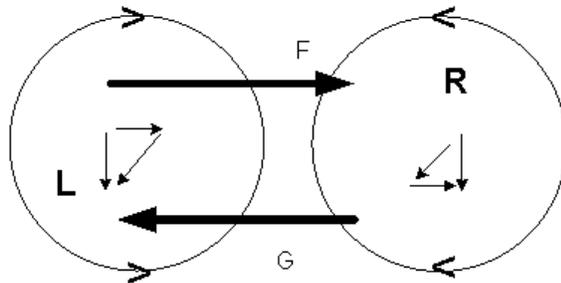


Figure 20. Adjointness between Two Systems

1. An information system has a left adjoint  $F$  i.e. right co-exact (physically represented) process which is uniquely co-ordinated by adjointness with a right adjoint  $G$ , left co-exact, underlying logical process  $F \dashv G$  as in the figure in Figure 3 reproduced here as Figure 20 [2, 1].
2. An ATM banking transaction where the left-hand triangle composes with delivery of cash simultaneously debited from the customer's account lacks a left-adjoint functor  $F \dashv G$  in a

universal Turing machine and therefore on any electronic digital computer with a von Neumann architecture. However a true quantum computer would not be so restricted.

3. Commercial practice seeks to model the adjointness with a sequential process that adheres to the principle of ACID.
4. The ACID principles which evolve out of good commercial practice can themselves be shown to adhere to a deconstructed monad, a triple over adjointness, as independently developed by category theorists.

## References

- [1] Freyd, P, *Abelian Categories, an Introduction to the Theory of Functors*, Harper & Row, New York (1964).
- [2] Freyd, P, & Scedrov, A, *Categories, Allegories*, North-Holland (1990).
- [3] Rossiter, B N, Heather, M A, & Sisiaridis, D, Process as a World Transaction, *Proceedings ANPA 27 Conceptions* 122-157 (2006).