The Universe as a freely generated Information System

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Abstract

The very accurate prediction of experimentally observed values of the coupling constants has meant concentration on the numerical *Combinatorial Hierarchy* in ANPA's 'internal process universe'. To understand further the structure and origins of the process of string generation itself needs the language of category theory. A formal representation of the database transaction can provide more insight into ANPA's *Program Universe*. The universe is an empty monoid. As an information system it needs to exist in physical form. Matter is generated in a non-local manner as strings by a natural process in the adjunction between 2-cells $F \dashv G$ where F and G are respectively the free and co-free functors.

1 Introduction

Irrespective of any mechanism like the big bang or steady state or bubbling in the latest bubble multiverse cosmology [20], there is always the question how the mechanism itself is to be derived. In category theory (CT) an origin corresponds to an initial object in a category and is given the label $\perp$ ('bottom'). The initial object of the universe as a category is the source of every other object in the universe – including the source itself, if recursive. In categorial terms there is nothing but arrows. Every object is an identity arrow, where the arrow domain and codomain are indistinguishable.

In a category that is cartesian closed there is a unique equaliser arrow [1] from $\perp$ to every object so the initial object of our universe

[1] See [23] at p.70 for *equaliser* and at p.97 for *cartesian closed categories*.

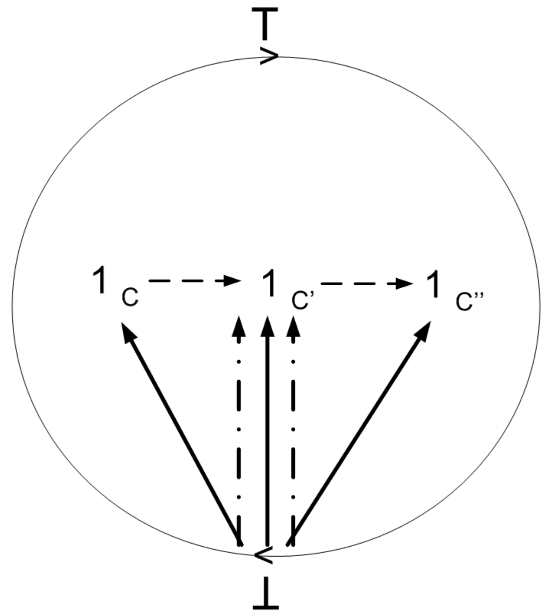

Figure 1: Initial Object.
By Composition with the broken arrows there is a unique arrow from the initial object to every object. The full vertical arrow is the equaliser of the two vertical broken arrows.

is the source of every object in it. It is cartesian closed [2] because we are only concerned with a universe that exists, that is with limits and exponentials. The exponentials are in effect all possible relationships (i.e. arrows) between objects. The equaliser is one type of limit and therefore a characteristic of cartesian closed categories. The equaliser is effectively a unique arrow between every pair of objects [3]. This is in accord with the usual definition of the universe as objects accessible to us. An inaccessible object is normally thought of as no interest to us. In physical terms the unique arrow is a resultant of all relationships. So the effect of one physical object on another is the resultant for instance of gravitational, electromagnetic and nuclear forces. However, the results of

[2]It should be noted for this paper throughout that we are concerned only with a constructive approach to reality, that which can exist. Therefore we do not need to go outside of the cartesian closed category nor resort to the category of sets. This simplifies the notation so that we do not need a gothic typeface to denote any category. In applied categories we are always operating in what a pure categorist might call formally a 'class' or enriched category.

[3]The unique resultant arrow is given in categorial terms by composition. This may be a composition diagram of two parallel arrows which as the vertical arrows in Figure 1 cannot be drawn as a triangle.

this paper suggest that a CT definition of the universe is a category of adjoint categories. The observer (if needed) is just such a category adjoint to all others and requires no special status. It also leaves open the question whether there are objects which cannot be perceived with the physical senses.

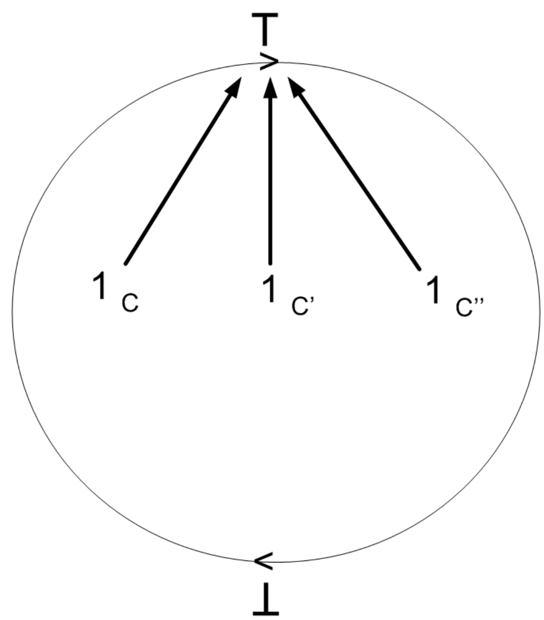

Figure 2: Terminal Object

Figure 1 shows a category that is cartesian closed and which therefore has its identity functor indistinguishable from its terminal object $\top$ ('top') and with unique arrows from its initial object $\perp$ ('bottom') to every other object. Other paths are possible as shown by the dotted arrows but any alternative path composes to the corresponding direct unique arrow. The closure of the category, $\top$, is depicted as a circular arrow because it is an arbitrary functor mapping the internal arrows onto themselves. The points are identity arrows of the category and not points in a mathematical space but much closer to the concept of a field. Figure 2 shows the terminal object with a unique arrow to it from each object. The terminal object is indistinguishable in a closed cartesian category from the identity functor, the intension of the category. The oppo-

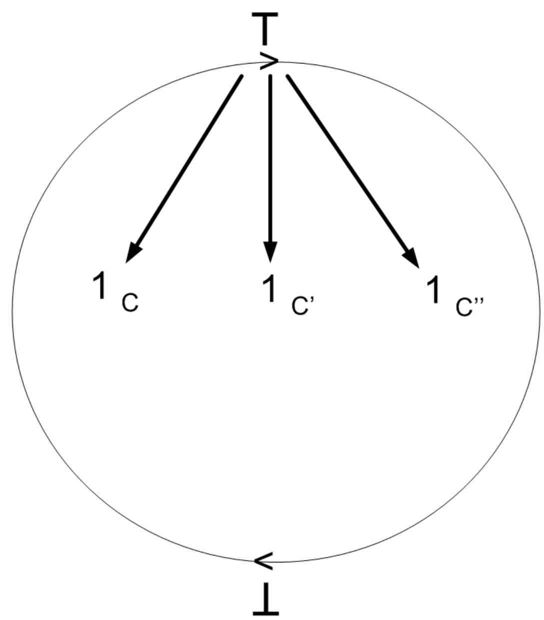

Figure 3: A category types its objects

site arrows in Figure 3 are therefore 'picking out' from the terminal object each internal object in the category, that is indicating that it belongs to that category in the sense that it is of that type. Note the general contravariant direction of the extensional typing [4] in CT as appears later. This was the seminal result of Lawvere [19] who was able to show contravariant functors to be bound up with intension/extension adjointness and logic quantification.

Earlier designations of the origin are traditionally zero (0) using number theory or the null set ($\emptyset$) in set theory. However neither of these are a true initial starting line because they are still under the starter's orders of preconditions like Peano arithmetic or the axioms of set theory. The set theoretic version of generating the universe is a hierarchy produced by taking iteratively the set of the null set and so on: $\emptyset, \{\emptyset\}, \{\{\emptyset\}\}, \{\{\{\ldots$. These can be ordered properly as a total order by inclusion but each label is representing a set at distinguishable levels and therefore of different types. There is no natural closure except by invoking some

[4]Those approaching CT from a set theoretic perspective often want to point a typing arrow from the object to its type but it is the other way round in CT.

platonic concept like infinity. This hierarchy treats every distinguishable object as being of a different type and therefore loses the very concept of type and any notion of natural categorisation. CT on the other hand recognises the typing and has a natural closure in a four-level sandwich of three interfaces. For in the first interface identity arrows together with arrows distinguishing them make up a category that is an identity functor in the second interface. Arrows distinguishing categories are functors and in the third interface arrows distinguishing functors are natural transformations. There this categorial cumulative hierarchy ceases because arrows distinguishing natural transformations are themselves natural transformations so providing natural closure. This sandwich is shown in Figure 4. As this is completely general and requires no assumptions other than the existence of the arrow this closure may be the ultimate explanation for the limitation within the four levels of ANPA's Combinatorial Hierarchy (CH).

$\longrightarrow$ Terminal closure with composition of natural transformations

Natural Transformations

$\longrightarrow$ Identity Natural Transformation

Functors

$\longrightarrow$ Identity Functors

Categories

$\longrightarrow$ Initial Identity Arrow

Figure 4: Sandwich of category theory layers

The universe itself is therefore a one object category consisting of internal transformations (i.e. arrows). Formally this is the structure of a monoid where the one object is both initial and terminal It is represented mathematically in set theory by a structure

$< M, f >$ where $f : M \longrightarrow M$ are permutation functions on the set M [5]. The one object is the intension and the internal transformation is the extension. Nevertheless, for natural numbers, it is perhaps worth noting set theory examples for natural numbers (N), the powerset $\wp X$ of the set X and strings of words (S^*) on an alphabet S:

$$\begin{array}{lll} \text{numbers} & < N, +, 0 > & < N, \times, 1 > \\ \text{sets} & < \wp X, \cup, \emptyset > & < \wp X, \cap, X > \\ \text{strings} & < S^*, \sqcap, \Lambda > & \end{array}$$

Categorically the central column represents colimits and the last column limits. Very significant is the gap for the limit monoid for the operation on strings, This is perhaps the discovery of Parker-Rhodes that there is such a fundamental operation as a process generator for the universe. In some sense the whole of this paper is concerned with this missing monoid involving an emergent process operation combining extension with intension. We need therefore to look in more detail at the concept of monoid, intension, extension and process. For this we find it convenient to go past the usual ANPA number theory CH and the generation of *Program Universe* to consider these as adjointness within an information system. A little inspection makes the monoidal structure very obvious. The first member of a triple is the underlying entity whether $\mathbf{N}$, $\wp X$ or (S^*) words on an alphabet S. The second member is one of the usual operations $+, \times, \cup, \cap, \sqcap$ (concatenation is $\sqcap$). The final member is the neutral element under the operation namely $0, 1$ the set X itself or the null string Λ.

In CT the monoid comes into its own with its full glory. It is introduced as early as page 2 of the *Categories for the Working Mathematician* of Saunders Mac Lane with the statement:

[5] see ([16] at p.66-67) where the standard *Handbook of Logic* in set theory gives the monoid only as an unnamed example although referenced *monoid* in the index.

> The notion of a monoid (a semigroup with identity) plays a central role in category theory

The structure $< M, f >$ from above is replaced by its much richer categorial version $< M, \mu, \eta >$ defined by commuting diagrams for μ and η. The significance for this paper is that μ looks back and η looks forward.

2 The Monoidal universe

The physical universe appears then to exist as an aggregate of extensional objects which can properly be formalised in CT as identity arrows. The objects are binary in the sense that the relationship between every pair of objects is reflexive and transitive. This structure is a preorder which is defined in CT in the sense that there is just one arrow between any pair of objects. A preorder is a one-object category where the binary relation defines internal subobjects. A category with a preordering is a semigroup as it is a binary operation [15, 21].

The empirical assumption-free universe consists of binary relations on potential objects. In pure mathematics this structure is an empty semigroup. However, the universe must have some identity to exist. An empty semigroup with an identity is a monoid. The intensional form of the universe is therefore a one-object monoid as already indicated. The extensional form of the universe consists of freely-generated internal objects – a free monoid. Internal objects of the universe can therefore be represented by strings generated from some sorts, that is an alphabet although atomic but not necessarily discrete. A singleton character in the alphabet $[s]$ maps under a free functor to a single character string $[s] \mapsto < s >$. A natural transformation compares the alphabet character with the character string $\eta : S \longrightarrow T(S)$ where T is a composition functor of an underlying functor G applied to some free functor F as shown in Figure 5.

$$
\begin{tikzcd}
S \arrow[r, "F"] \arrow[d, "T"'] & F(S) \arrow[dl, "G"] \\
S^* &
\end{tikzcd}
$$

Figure 5: The whole diagram is a natural transformation arrow namely $\eta : S \longrightarrow T(S)$ representing a free/co-free structure

The old word problem was to define on a given alphabet S all possible concatenations of finite strings S^* i.e. words from the given alphabet. S^* is sometimes known as Kleene closure.

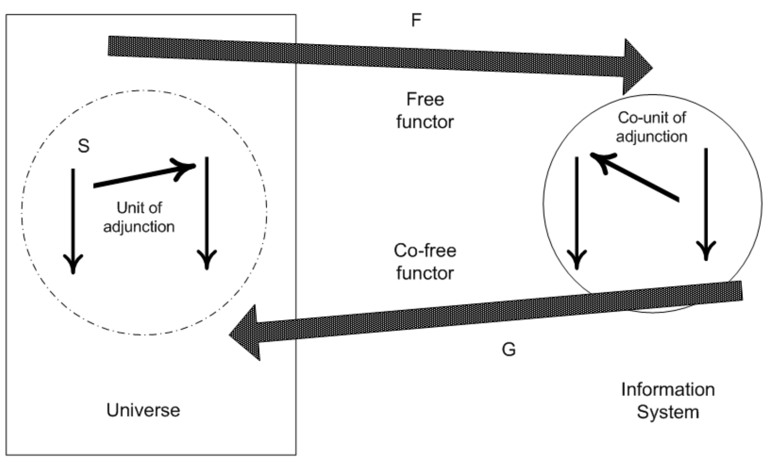

Figure 6: The View of the universe as an Underlying Information System

A tenet of ANPA is that the universe has come about from the generation of strings from something usually described as nothing. The strings form natural structures and it has been observed by Parker-Rhodes [26] that by assigning simple number bits to the strings [6] their calculation in a natural CH correlates very well with fundamental coupling constants of the universe [6]. This process has been named *Program Universe* [24]. It is the analogy of the computer program to suggest that on each cycle of the computer's internal clock, a string is generated. This builds up an information system populated by the generation of new strings distinguishable from those already generated. It is an 'internal process view' [3].

3 Intension and Extension

Intension and extension are used here in their general sense of comparing a fundamental class of identity relationships. The full abstract definition needs CT. This we have provided in Figures 1-3 above. In the case of sets the concept of membership is fundamental. A set may be identified by some label that connotes its intension e.g. 'Greek Alphabet' or by a denotational extension with elements consisting of the characters of the alphabet $\{\alpha, \beta, \gamma, \ldots, \}$ [7]. This relationship will then be relevant in any application of set theory. For example Codd's relational database model relates tables of data with an intension such as a name like 'author' where the extension consists of a pool of data values like {'Marlowe', 'Dickens', 'Shakespeare',…}. This example shows the 'elementary' limitations (that is the simplicity of elements) in set theory. There is no formal relationship between the intension other than the connection between the words 'alphabet' and 'characters' (that relationship in natural language does have a formal basis in the sense of strings that we shall come to shortly). Nor is any formal relationship, either inter-elements or intra-elements that may exist, made explicit. Nevertheless they are assumed to be disjoint.

[6] Recent work suggests that this correlation for the fine structure constant is exact to seven significant (decimal) figures [17].

[7] The convention adopted of writing a list like this as a sequence identifiable from the labels used can be misleading. Extensional elements of a set have no order.

In the example given there may be interrelationships between author names like Charles Dickens and Monica Dickens and much more complicated relationships (at the pragmatics level) between Marlowe and Shakespeare. There may be relationships within an element as when a book has more than one author. These interrelationships are outside strict set theory. Names can be coined like multisets or bags but these are really all bastard forms not part of any consistent theory.

A little more sophisticated intension/extension relationship is between the closure and the interior of open sets in topology. This, because of the relationship between intension and extension, has much more power for applications [8] for it gives relationships both within data and between the extension of the interior and the extension of the closure. Other examples are the use of instantiation where there is a suggestion that the intension is the codomain of some morphism from intension to extension and 'meaning' which goes the other way from extension to intension. A recognition that the relationship may take various forms is to be found in the object-oriented paradigm with notions like encapsulation, polymorphism, etc. In the elementary version of set theory both extension and intension are fixed. Because this restriction is not very realistic (i.e. not found naturally in reality) many attempts to relax this limitation can be found although there is seldom any attempt to justify these variations formally.

Very significant in this context are Gödel's celebrated Incompleteness Theorems and Church's undecidability result ([27] at p.599). By Gödel's first theorem extension is undecidable for axiomatic systems with arithmetic and intension likewise by his sec-

[8]For instance the Kuratowski postfix unary closure operator (defined by $A\sim\ \sim = A\sim$ and $0 = 0\sim$) was used in an early definition for consciousness by one of us [8] at p.287 in ANPA9 proceedings but we have since upgraded our ideas on consciousness more on the lines of this present paper [10], [11], [13]. Kuratowski closure can also provide an alternative set-free foundation for toplogy.

ond theorem. Examples that are capable of escaping the clutches of Gödel are the Galois connection and language. If built of fabric more general than sets these will be able to exhibit the kind of behaviour to be described in the main part of this paper. The example of language is especially important on account of its much more elaborate version of intension/extension. The structure of the intension language is syntax while the structure of its extension is the semantics. These (particularly the latter) are expressed using sets as for instance in the theory of computer programs. The great power of natural language (but not usually of artificial languages as computer programs and other comparable modelling techniques) is that there is a third layer with syntax, semantics and pragmatics where the pragmatics process as the context sensitivity of the real world.

It is not perhaps surprising that the universe has the most elaborate structure of all with the three levels of syntax, semantics and pragmatics for both intension and extension. Three levels raised to the power of two give classically a bi-cubic, as the signature of a full information system. This is the underlying structure of the double helix of the DNA and is also of the same order as the Kortweg de Vries equation [9], the generalisation of the Schrödinger equation. The universal closure (i.e. intension) is achieved with at the most three interfaces each interface consisting of an intension/extension relationship [28].

3.1 The Arrow as Process

The dimensionality of three interfaces is present in set theory by taking the intensional and extensional forms as discussed above together with functions between sets. There is a difficulty. There is no formal connectivity between the set intension and its extension. Functions relate sets but are external. With arrows on the other hand the three-dimensions are intension, extension and the direction of the arrow and all three are internal to the concept of a cartesian-closed category. This means that the arrows can better represent a concept of process. It is one of the objectives of ANPA to study the universe as process and what results from process. Bastin [3] enunciates seven principles in support of the universe as process:

- I. Process As A Necessary Principle
- II. Cumulative Sums
- III. Iteration And Algebraic Structure
- IV. Program Universe
- V. Perception
- VI. Iteration, The Statistical Background
- VII. A Parallel Development

It is illuminating to compare these headings with the arrow of CT as a process. In his first *Process As A Necessary Principle*, Bastin describes the construction

> to express algebraically the construction of successive new sets of entities out of the operations upon the elements of a previously existing set.

CT builds a geometric logic on the Bourbakian tripartism of algebra, topology and order. It still has the algebra expressed as strings but also has the further properties of topological openness and the built-in concept of order arising from the direction of the arrow. We shall see below how the arrow upgrades Bastin's algebraic constructivism to a geometrical construction of a succession of new categories of objects.

Bastin justifies his second principle of *Cumulative Sums* as follows:

> We soon found it necessary to see the constructed elements as discriminately closed subsets. To get the numbers right for experimental identification it was necessary to add those of the different stages together.

In the categorial version this adding together of stages corresponds to a colimit of limits. Iteration gives levels of an hierarchy where an intension generates an extension which names a new intension. This intension generates a further extension which adds to the last intension. Therefore we get a categorial CH consisting of colimits and limits. A limit is 'discriminately closed'.

For *Iteration and Algebraic Structure* Bastin comments

> So the hierarchy algebra appears as a set of rules which constrain the development but do not prescribe it.

This categorial version provides both for freeness (unprescribed development) and for co-freeness (the prescription of rules) formally integrated in an adjunction [9].

Bastin queries the basis of *Program Universe* and whether it can be a model:

> Was it an algebraic device merely or did it have a counterpart in the world? Sometimes it was said to be just a model, but if so what was it a model of?

The answer from CT itself seems to be that it is not a model but a reality of which the universe is an instantiation. The CH in its classical ANPA description is then a numerical model of this reality.

[9] For freeness and co-freeness see below.

The fifth principle of *Perception* means to Bastin that the generation of the CH is a construction and not a matter of filling a platonic receptacle viewed by a passive observer. The observer is then part of the construction and deconstruction in the iteration. The arrow of CT well represents this viewpoint and does not require an underlying mathematical space nor the concept of a vacuum. The arrow provides everything the vacuum provides and more. The split idempotent [10] generalises the concept of vacuum.

In his sixth principle, *Iteration, The Statistical Background*, Bastin points out the need for some understanding of 'statistical' and 'random' concepts. Statistics are right-exact concepts in CT and subobject classifiers of a topos while randomness is part of the freeness/co-freeness principle.

An appeal to *A Parallel Development* by Cahill on 'quantum foam' is made by Bastin in support of the concept of process as his seventh principle. Cahill relies on Leibniz' monad as the basic unit with the nature of a gebit (a pre-geometrical bit). This appears to be the same notion as found in the geometrical aspect of the categorial arrow where the monad induced by an adjunction can be identified with the monad of Leibniz ([14] at p.308). The arrow goes further than Leibniz and even subsumes Aristotle's comparable fundamental particle, the *entelechy* which unlike Leibniz' monad has an inbuilt direction pointer. Cahill's words:

> Process physics is a semantic informational system and is devoid of *a priori* objects and their laws and so it requires a subtle bootstrap mechanism to set it up

as cited with approval by Bastin (within a longer quote) might well have been used to sum up the categorial mechanism that fills the second half of this paper. From the rest of the quotation we see that Cahill relies on square matrices to express relational informational

[10] See [23] at p.20.

strengths in a stochastic neural network. Matrices as operators are of course the counit 'bits' of a functor in CT.

3.2 Generation of the Physical universe

It is the purpose of this paper therefore to amplify in a formal manner using CT some detail not present in the traditional presentation of the CH. One important point is to justify the generation of physical matter. In the ANPA descriptions there are two levels, the process and the data corresponding to intension and extension. But also the process is the data. *Program Universe* relies on a classical von Neumann paradigm where the process is some algorithm that operates on the data but there is a mixing together in some string of words as in a high-level language or as a bit stream in an assembler language. The database approach promoted by the ANSI/SPARC standard treats the program (usually an algorithm) as quite distinct and each may be stated independently with the programmer's meta data and data having the property of persistence. Neither seem to comprehend the spirit of the ANPA philosophy though the process is data at the same level as the data. Proponents of the ANPA approach have so far concentrated on only some aspects. Very little is available on the mechanism of the progress but it appears generally to rely on a set theoretic perspective with generation from the null set. More attention is paid to discrimination where there is process upward in a particular natural hierarchy of bit streams with levels filled by discrimination against strings already generated lower in the hierarchy. In database practice this is a generate, search, look-up, test and store recursion. While a set theoretic approach to the CH based on natural numbers has been able to provide some very compelling results consistent within a very wide range of experimental data [25], nevertheless the discussion here suggests more formal underpinning is needed for the representation of 'process' and in the way extension merges with intension in 'discrimination'. CT on the other hand is able to give some support in these areas to investigate the arrow version of the *Program Universe*. The analogy of the tick-tock clock is that the whole universe turns over in some discrete fashion from one configuration to the next in some quantum space time frame. The advantage of using CT is that we are not restricted to the limitation of set theory and are not excluding possible results of quantum mechanics and especially the general and special theory of Einstein's relativity. Our approach is perhaps to make more general that of [2] which has already yielded a statistical and algebraic alternative to classical and quantum space and time.

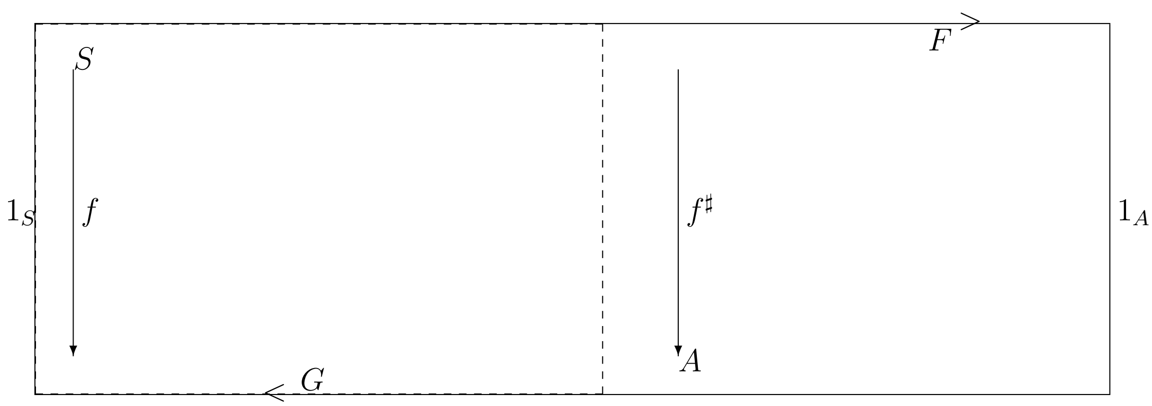

Figure 7: Correlation between Arrow f in **S** and $f^\sharp$ in **A**

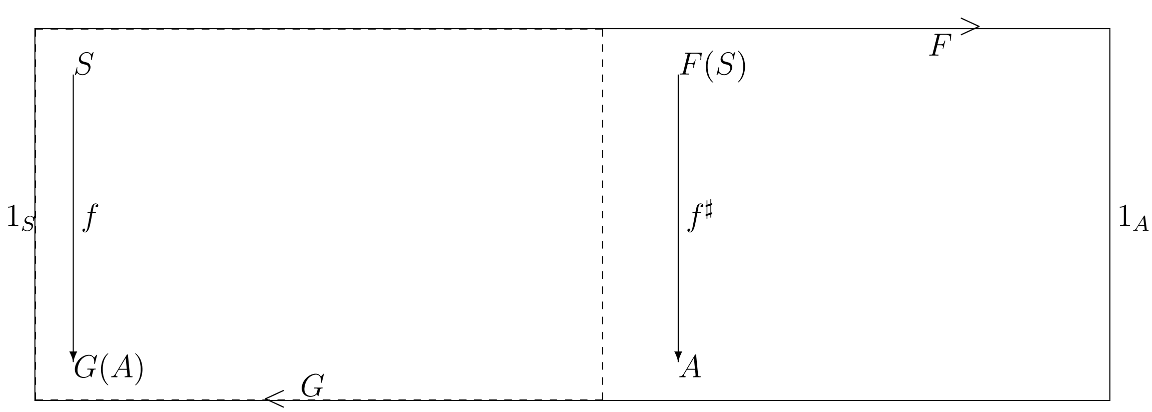

Figure 8: Identity of the universe (intension): Correlation between Arrow f in **S** and $f^\sharp$ in **A** where $\eta \longrightarrow \perp$

The example of Codd's relational model shows that the intension/extension relationship is rather obscured by the flat nature of sets without integrated functions. In categories on the other hand the focus is on arrows integral with objects and where even the objects are just (identity) arrows. Arrows between arrows give levels which are not easily identifiable in sets. In the theory and practice of databases the arrows are fundamental but they are often described by words such as 'relationships', 'methods', 'stored procedures', 'functional dependencies' or 'normalisation'. Often what are significant are arrows between arrows which is the essence of typing and therefore the fundamental use of a domain as typing. Transactions are important everyday use of database features involving arrows between arrows.

A transaction is a dynamically structured process. For instance a straightforward banking transaction requires a sophisticated relationship between crediting and debiting with resort to a fail-safe procedure. The application of everyday business rules involve interaction between intension and extension. For instance an ATM withdrawal for a class of customer may be limited to a specific value like 300€ per day. There is a strong physical component in this transaction. The customer's account cannot be debited until the bank notes have emerged from the hole in the wall in case the transaction fails to complete because of some mechanical failure. On the other hand there has to be a certainty that the amount will be debited once the money has been withdrawn despite any failure in the electronic process. This is achieved in practice by adherence to the ACID principles [11] with every withdrawal of cash being written to a transaction log before the money is paid out. Effectively the transaction log is written up prospectively in advance to a secondary file and then in the event that a particular transaction fails

[11] ACID stands for Atomicity, Consistency, Isolation, Durability [5].

to complete, it can be unpicked later by re-running from the last successful transaction to undo the steps in the log that were never fulfilled. Physical recording aspects of data in hardware are usually on disk for persistence. However, if the whole transaction were to be carried out electronically in an e-banking transaction there would still be some physical involvement because the transaction has to reside somewhere such as the hardware of the main memory. This is an example of the principle of Landauer [18]. Information cannot exist except in the physical form. The logic of an empty monoid is itself information and must be manifest in material form. This is therefore the explanation of matter in the universe.

There is also another aspect which shows up in this banking transaction, that is parallel processing. The purpose of the separate log is to provide an overlapping alternative resource in case of breakdown in the main transaction. True parallelism of the simultaneous recording of the transaction with its performance is not possible in a von Neumann architecture which relies on a sequence of processes between fixed cells. This is because classical computation is local [14] and is the reason for the failure of initiatives in the 1980s in parallel processors and the limitations of set theoretic (and therefore local) models like Petri Nets. The universe itself on the other hand is non-local processing and therefore can carry out simultaneous events although communication between them is not possible because that imports localisation. This is manifested in the cosmological limits like the finite velocity of light and the issues of the special relativity that arise from it. The operation of a separate log succeed as a way round the problem by providing two real-time systems [12]. Because of this there is no absolute time

[12]This is real-time in its true sense namely this is the origin of time. Real time is used here in the technical sense. Real time is the sequence of operations of a system. This may be synchronous as with some clocks or asynchronous. This defines a particular inherent time system, based on intensional time. So there are different possible systems of time. They have to be related through their extensions, for instance the difficulties of relating crystal time with sideral time or solar time as determined by the

on the von Neumann machine for serial processing with fixed cells which determine how a choice is made. This is a weakness of the *Program Universe* as a model and justifies the fuller explanation in information systems available in a database transaction for the generation of bit streams.

This banking transaction is typical of any transaction as a non-local process. The log provides a parallel information system to effect the banking operation consistently. It has two components, one looking forward to the sequence and one looking back to check that what was expected was achieved. The universe carries out transactions all the time non-locally mediating between objects in time and space. It nevertheless still has the forward and back components except that they are non-local. This emerges in the following analysis of adjunctions in CT. The universe operates as a quantum information processor [13].

4 Process as a Semantic Information System

A feature of a system may be a state, an action, a process, a property, indeed anything the system is or does. Outside of CT any of these are usually represented by a set, that is with unordered elements or with some imposed order like a vector or tuple. The ordering is independent of the notion of a set. In CT any feature of a system is an example of the arrow. The direction of the arrow already includes the notion of ordering and also has inherent typing so that a feature of a system is naturally distinguishable.

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rotation of the earth. This is the essence of time as a local phenomenon projected out of space time under application of the axiom of choice.

[13] The banking transaction is a type of process very suited to the quantum computer. It is for this reason we have been urging recently the quantum processing of information and evolvable databases as a more realisable everyday application than some of the sensational and more esoteric examples being promoted like code breaking, teleportation and remote viewing [29, 30].

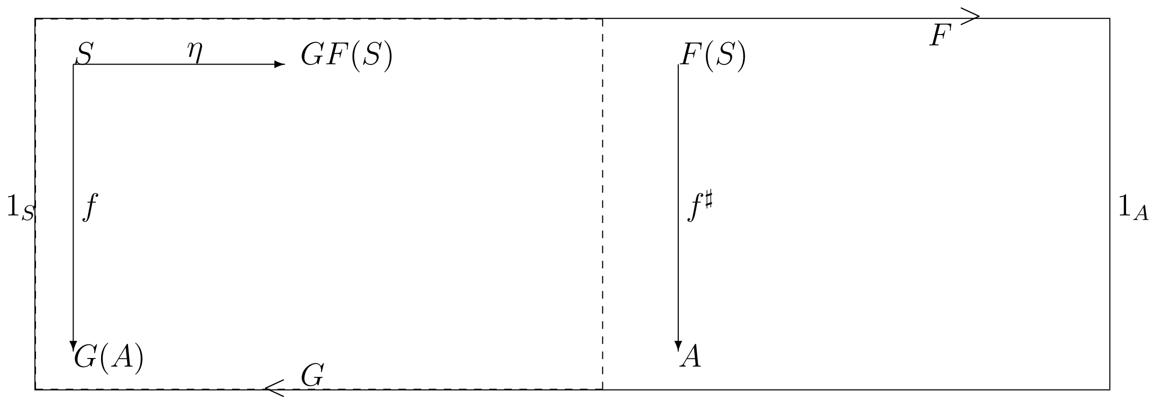

Figure 9: Working of the universe (extension): Correlation between Arrow f in **S** and $f^\sharp$ in **A** where η other than $\perp$

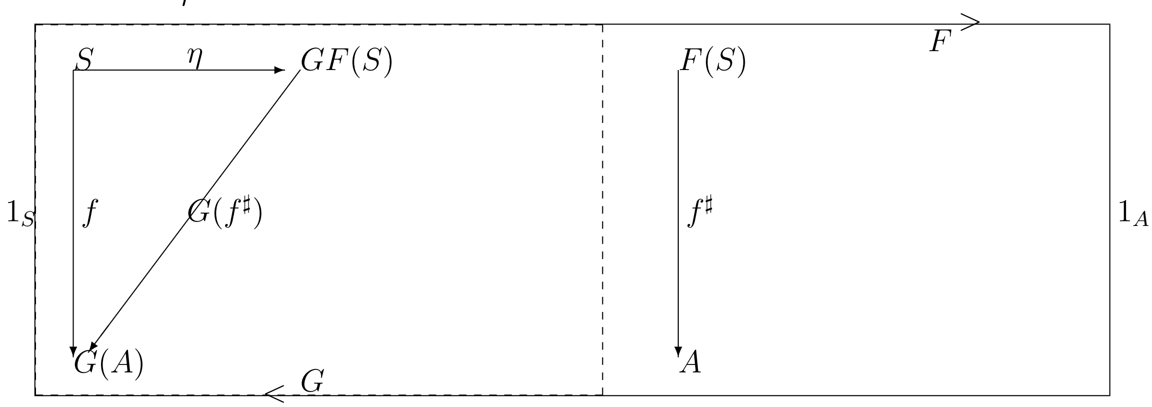

Figure 10: Distinction of $f^\sharp$ in **S** by arrow $G(f^\sharp)$

The CT information system instantiated as the universe is depicted in the Figure 6 and elaborated in the sequence of figures that follow. The instantiation as the one-object monoidal physical universe comes about as explained because natural logic is information and therefore exists in material form. Each of the figures is a topos with the usual properties that can be found in Johnstone [15] (*passim*) or modelled in higher operads of n-categories [21]. Fortunately applied categories can be restricted to the deep simplicity of nature and we need no more than the fundamental properties of adjointness that can be found in any basic textbook [14]. However, to bring out the dynamic structure of process that is intrinsic in adjointness we will set out the behaviour of the arrows step by step. The left-hand category (S) in each diagram is

[14] An example is the second edition of *Categories for the Working Mathematician* [23].

the monoidal universe. The functors F and G between S and the right-hand category (A) are endofunctors so both S, A categories are really coincident on the left but drawn side-by-side to make the relationship more patent for the overall transaction of Figure 6.

In more detail Figure 7 shows a typical arrow on the left (f in S) which is a family of arrows that correlates with a family of arrows in A which are represented in the figure by a typical right hand arrow $f^\sharp$. Correlation under adjunction is given by

$$\frac{\eta : \mathbf{1}_S \Rightarrow GF}{\overline{\epsilon : FG \Rightarrow \mathbf{1}_A}}$$

The double bar indicates implication and its converse. GF is the functorial composition of applying functor G to the result of applying functor F to category $\mathbf{S}$. FG is the corresponding application of functor F to the result of applying functor G to category $\mathbf{A}$. Both arrows above and below the double bar could be replaced by the usual symbol $\leq$ for reflexive transitive ordering, an example of an arrow from where the ordering is derived as mentioned above.

The unit of adjunction is $\eta : 1_S \longrightarrow GF$ and the counit is $\epsilon : FG \longrightarrow 1_A$. If $\eta \longrightarrow \perp$, GF returns the arrow f to its original state f. That is F maps object S to $F(S)$ as G maps A to $G(A)$ as in Figure 8. If η is other than $\perp$, functor G will take $F(S)$ to a different object in S. So we have $\eta : S \longrightarrow GF(S)$ in Figure 9. Note the distinction shown in Figure 10 of $f^\sharp$ under functor G as the arrow $GF(S) \longrightarrow G(A)$ labelled $G(f^\sharp)$. Because of the uniqueness of adjunction there will be only one possible arrow $G(f^\sharp)$ given by the composition of the triangle shown in Figure 11.

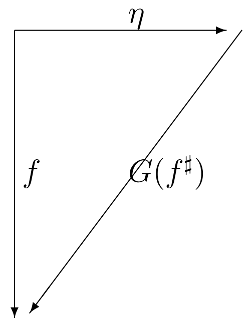

Figure 11: Uniqueness of adjunction: only one possible arrow $G(f^\sharp)$

Figure 10 is the explanation of naturality [15]. What happens to the arrow whose source object is $GF(S)$? In this case we have the dual perspective, representing co-freeness as shown in the following Figures 12 to 14. If $\top \longrightarrow \epsilon$, FG returns the arrow $f^\sharp$ to its original state $f^\sharp$. If $\top$ is other than ϵ, functor F will take $G(A)$ to a different object in A. So we have $\epsilon : FG(A) \longrightarrow A$ as in Figure 12. Note the distinction in Figure 13 of f under functor F as the arrow $F(S) \longrightarrow FG(A)$ labelled $F(f)$. In Figure 14 we introduce the correlation between an arrow g in $\mathbf{A}$ and $g^\flat$ in $\mathbf{S}$.

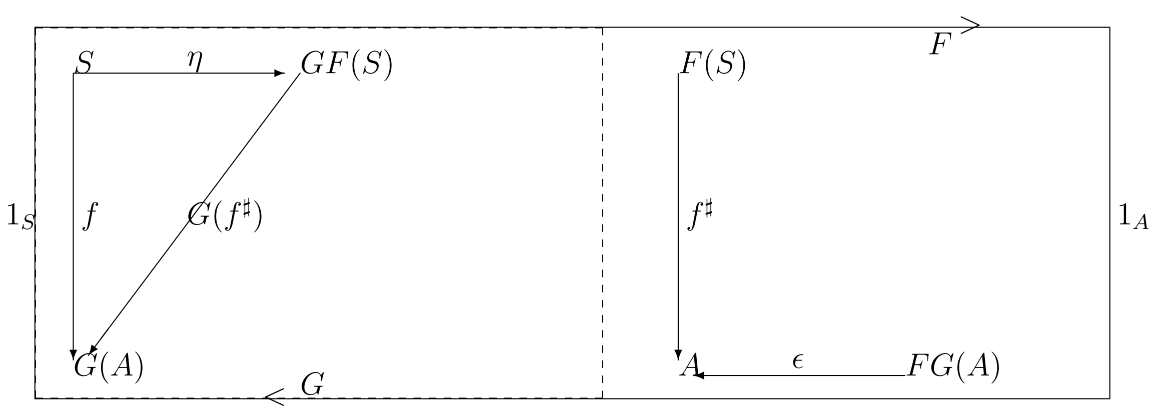

Figure 12: Correlation between Arrow f in $\mathbf{S}$ and $f^\sharp$ in $\mathbf{A}$ where $\top$ other than ϵ

In Figure 15 we show the mappings that occur in correlating g in $\mathbf{A}$ with $g^\flat$ in $\mathbf{S}$ where η is other than $\perp$ and $\top$ other than ϵ.

[15]Although not justified here this naturality is defined in the paper [28].

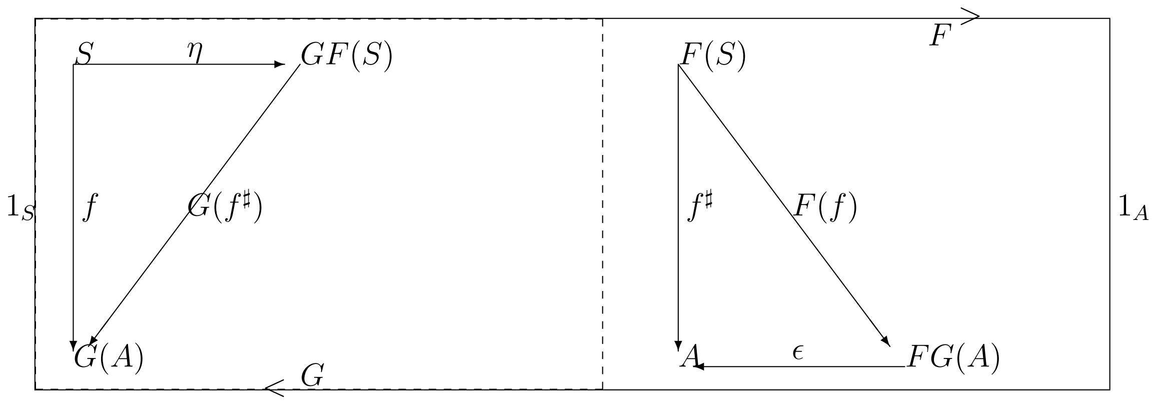

Figure 13: Distinction of f in $\mathbf{A}$ by arrow $F(f)$

In this diagram we have a general relationship where neither truth nor falsity hold [12]. The complete picture of the adjointness is given in Figure 16 to illustrate all the relevant mappings between an arrow f in A and another arrow g in S where η is other than $\perp$ and $\top$ other than ϵ.

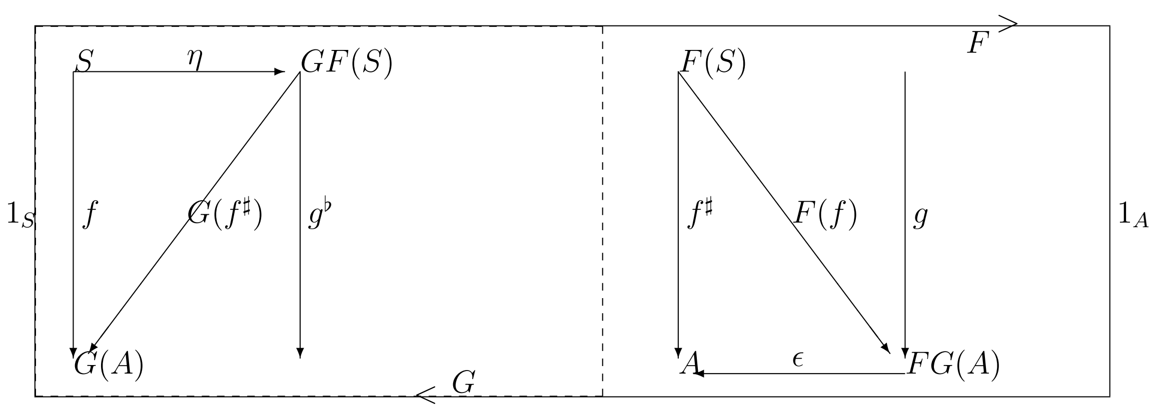

Figure 14: Correlation between Arrow g in $\mathbf{A}$ and $g^\flat$ in $\mathbf{S}$

Figures 7 to 16 show in detail the nature of adjointness, in a manner perhaps more suited to implementation in a computer system than is the normal approach with CT in mathematics where abstraction is usually preferred. The build up is from arrows in $\mathbf{S}$ to correlating arrows in $\mathbf{A}$ for representing the freeness associated with the free functor F. The co-free functor G is the underlying functor which is critical in establishing how well $\mathbf{S}$ reflects $\mathbf{A}$.

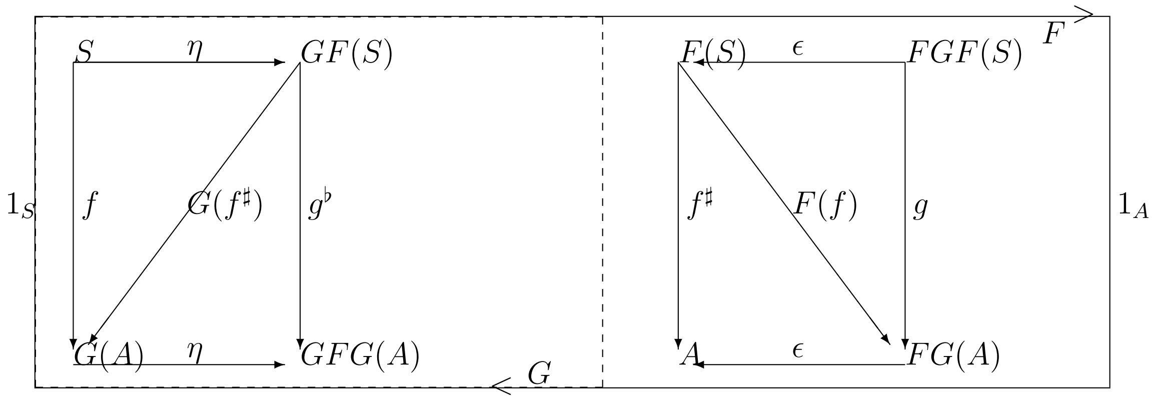

Figure 15: Correlation between Arrow g in **A** and $g^\flat$ in **S** where η is other than $\perp$ and $\top$ other than ϵ

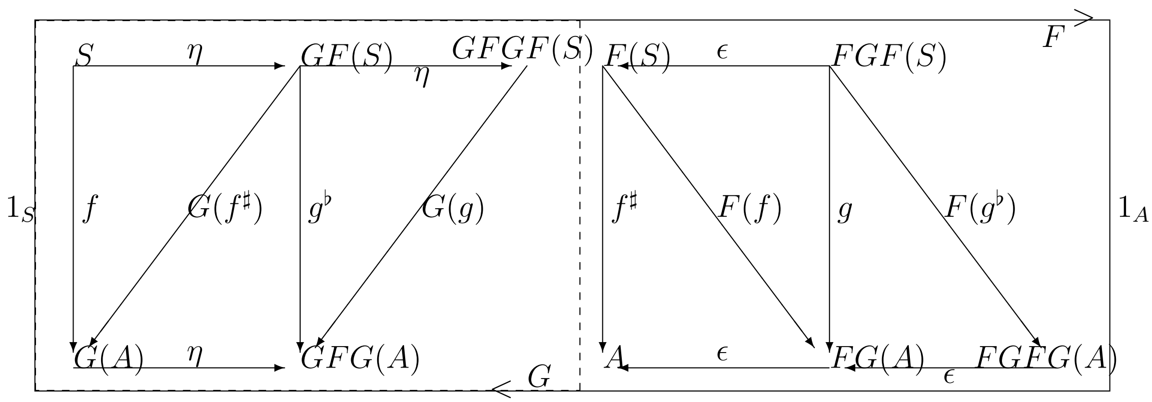

Figure 16: Complete Picture: Correlation between η and ϵ in 2-cell Adjunction $F \dashv G$

5 The Contravariant Intension/Extension Mapping

So far we have considered adjunctions which are covariant, that is domains of arrows in one category are mapped to domains of arrows in the other category. Similarly codomains in one category are mapped to codomains in the other category. The covariant case will apply across a single level such as mapping from one intension to another or from one extension to another. If we adjust our diagram in Figure 6 so that the left-hand side is the intensional universe and the right-hand side is the extensional universe, then the mapping will be contravariant. The result is shown in Figure 17 in which the arrows in the left-hand category have been reversed. In multi-level mappings, such as intension/extension,

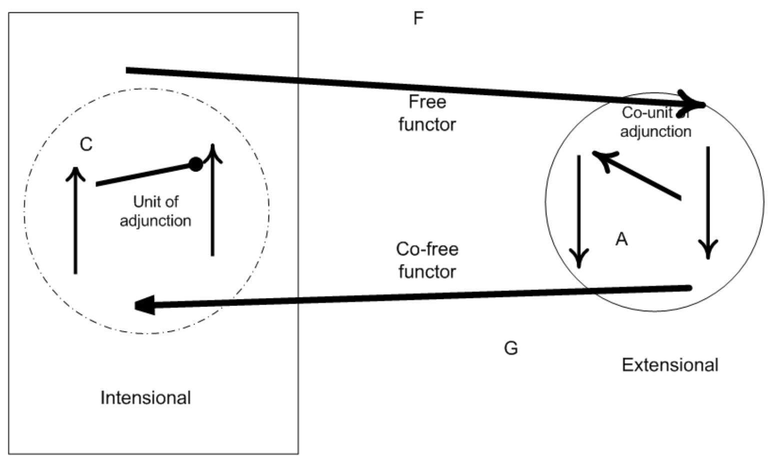

Figure 17: Contravariant Mapping between Intensional universe (left) and Extensional universe (right)

the relationships have long been known to be contravariant [19]. With a contravariant functor, domains and codomains in one category are mapped to codomains and domains respectively in the other category. In information systems the extension is of the form *value* $\longrightarrow$ *label* and the intension is of the form *label* $\longrightarrow$ *type* so that the relationship between them must be contravariant if one is to be mapped on to the other.

The diagram in Figure 17 is not unlike that for interhuman communication proposed by [22] which assumes two levels are involved. At the first level information is exchanged and provided with meaning, and at the second level meaning can reflexively be communicated.

The formal diagram in Figure 18 shows arrows reversed on the left from those in Figure 16 except for η which still compares S with $GF(S)$ and $GF(S)$ with $GFGF(S)$. This diagram shows the intensional universe on the left and the extensional universe on the right. The Galois connection [31] can be used to reason with such diagrams.

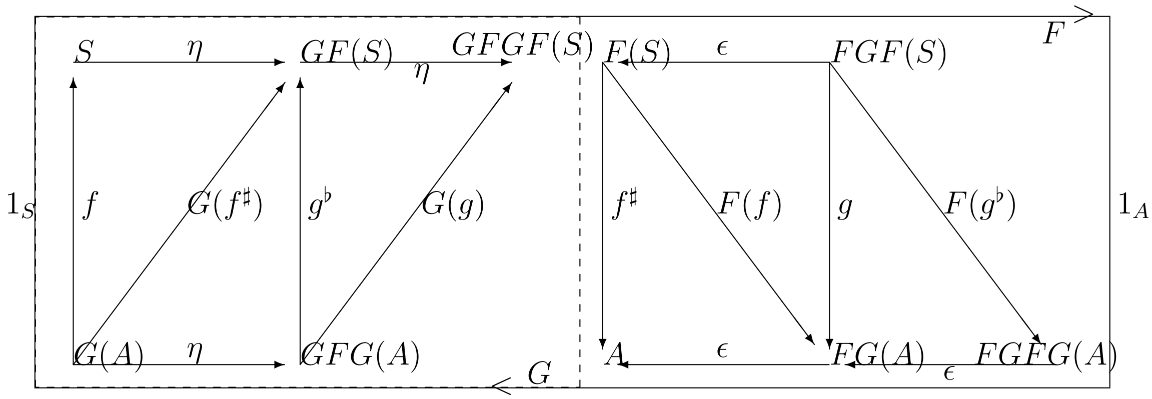

Figure 18: Complete Picture: Correlation between η and ϵ in 2-cell Contravariant Adjunction $F \dashv G$

6 Abstract Representations of Covariant/Contravariant Functors

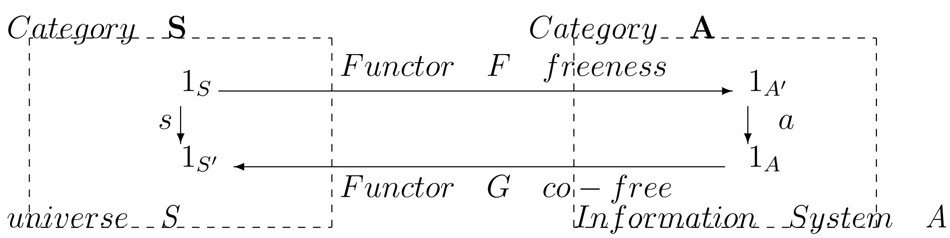

Figure 19: Covariant Mapping between universe and Information System: $\eta \longrightarrow \perp$ and $\top \longrightarrow \epsilon$

A more abstract representation (extending that in [14]) is shown in Figures 19 to 21. Figure 19 corresponds to Figure 8 where the unit of adjunction $\eta = \perp$, giving a simple equivalence between the two categories **A** and **S**. Figure 20 corresponds to Figure 10 where the unit of adjunction η is other than $\perp$ with η taking S to a different object $GF(S)$. Figure 21 corresponds to Figure 13 where for the counit of adjunction, $\top$ is other than ϵ, with ϵ taking $FG(A)$ to a different object A.

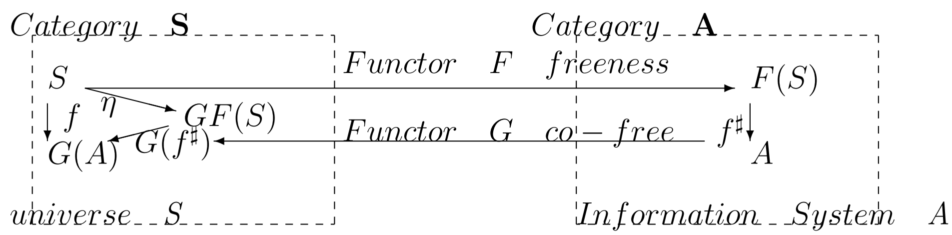

Figure 20: Covariant Mapping between universe and Information System: unit of adjunction η other than $\perp$

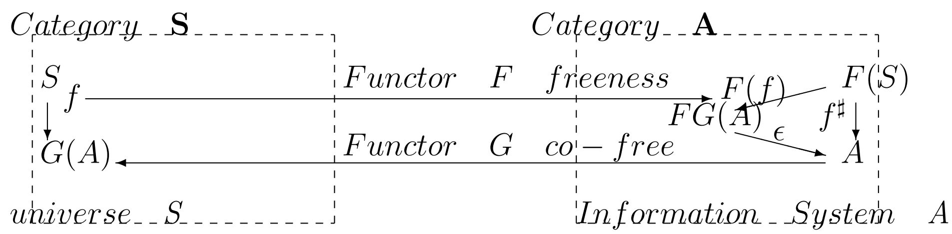

Figure 21: Covariant Mapping between universe and Information System: counit of adjunction $\top$ other than ϵ

Figures 19-21 show the covariant mapping between a universe and an information system. If we consider that category **S** is the intensional form of the universe and that category **A** is the extensional form of the universe, then the mapping between them will be contravariant as discussed earlier. Each of Figures 19-21 can be represented in contravariant form by reversing arrows, other than η, in the left-hand category **S**. Reversing the arrow $s : 1_S \longrightarrow 1_{S'}$ in Figure 19 has an apparently trivial effect as the type of the domain and codomain are the same. However, trivial effects may be of greater significance in applications than in pure mathematics. When η is other than $\perp$ or $\top$ other than ϵ the effects of reversing the arrows in category **S** are obviously of greater significance. Here we show in Figure 22 the contravariant form of Figure 20. This shows the reversal of the directions of f and $G(f^\sharp)$ with the commuting triangle now giving the equation $\eta \circ f = G(f^\sharp)$ instead of $\eta = G(f^\sharp) \circ f$ as in the covariant form. The arrow $F(S) \longrightarrow A$ in **A** is mapped by G on to $A \longrightarrow F(S)$ to give $GA \longrightarrow GF(S)$. This is contravariant as the domain and codomain of **A** are mapped on to the codomain and domain respectively of **S**.

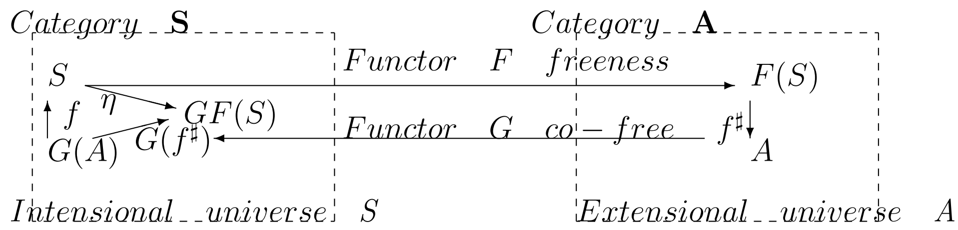

Figure 22: Contravariant mapping between intensional universe and extensional universe: unit of adjunction η other than $\perp$

For the contravariant form, if indeed $\eta \circ f = G(f^{\sharp})$, then the diagram is natural. Further when η is other than $\perp$ then the arrows are distinguishable by the application of GF to S which returns an arrow $GF(S)$ which may be different to S. The combination of contravariance, naturality and distinguishability provides a formal basis for relating the intensional and extensional universes.

7 Summary and Future Work

Let us take stock by summarising the argument. Firstly there is the monoidal universe in which the intensional universe is portrayed as a one-(object) monoid and the extensional universe as a free monoid freely-generated internal objects. This provides for the generation of strings from what is usually described as nothing.

The relationship between the universe and its representation in an information system has been represented in its most general form by covariant adjunctions, which have been built up in detail in a series of stages. When the relationship is considered instead between an intensional universe and an extensional universe, then the adjunctions are contravariant to handle the two-level intension-extension mapping. More abstract formal, natural diagrams have been developed to show both the covariant and contravariant adjunctions.

Relevant aspects not fully pursued include time dependence. This is because we are dealing with a non-local condition where neither time nor space are to be explicitly differentiated. It would not be difficult to add time if it was needed. We have shown elsewhere [12] that it is just the matter of making every time-dependent category a slice category. It appears that time is not even needed for local conditions. Contrary to earlier suggestions [7] recent high quality data from ESOs Very Large Telescope array in Chile show no evidence to support a time variation in the fine structure constant [4]. Also we have not dealt with pragmatics but that is just the need to provide context sensitivity by adding an ambient category. This is achieved by enriching every category with pragmatics to a topos.

On the other hand it will be of great interest to pursue further the emergence operator referred to at the beginning of this paper to show if it is some fundamental characteristic of a relationship between intension and extension as the phenomenon of 'discrimination' in the theory of ANPA's CH seems to suggest.

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