Logical Monism: The Global Identity of Applicable Logic

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Abstract. 'One universe, one logic' takes the world as it is and leads to adjointness as the global logic of anything. The alternative approach to find a unification of known logics requires assumptions and is therefore consistent with the same conclusion for a universal logic has to be universally applicable. The universal characteristic of adjointness is that it has a natural construction from the concept of the arrow. The application to the test sentence, 'John said that Mary believed he did not love her', demonstrates adjointness as the logic of the post-modern world.

1 Unity of Applicable Logic

There is one ultimate logic: it is a simple ontological but pragmatic argument of 'one universe, one logic'. If more, how can we know unless there is a logic to compare them? If logic is a family of varying strength, what logic compares the variance? Only some ultimate logic. How do we even know this? It must still be the same logic that tells us this. And that logic must tell us about itself -- tell us that it has some recursive self-closure. The same pragmatic cogency leads us into the world of physics and beyond into the humanities. The world must fit together according to this same ultimate logic. It is therefore an applicable logic. Universal logic means universally applicable logic. This study arises from the investigation of fundamentals in two large applied areas: one is schema design in interoperable databases, the other is in legal reasoning; both studies relate logic to real-world facts. Until we are able to identify the ultimate logic of the universe, it is not surprising that goals like unified field theory within a "theory of everything" are so elusive.

Applicable logic is needed in new ways in biology, medicine, economics, legal science, natural computing, modern physics, etc. This means it has to be a logic which can manage the advances made in the twentieth century, many of which are not amenable to classical logic. There is a whole class of new problems that have arisen mainly concerned with the logic of information that is needed in modern biology, medicine, business, the humanities, etc where classical logic has very limited application. There are various branches from classical propositional and predicate to modern logics like intuitionistic, many-valued, relevant, paraconsistent, quantum, modal, polar, nonmonotonic, linear, lamdba, pi (open network), etc [3]. To be valid these will be models or views of universal logic.

There is clearly a logic inherent in physical processes. So the laws of physics need to be satisfied including those of quantum physics. Beyond physics is biology where life sciences may have a logic that at the end of the day cannot conflict with the laws of physics even though life itself may not be obtainable directly from physics and its laws. Certainly the continuity of the kind of logic to be found in the structure of the genome and genetic processing must be part of any one single logic.

However if this one logic is to be fundamental to the universe it must also satisfy everything else we know that goes on in the universe like human activities. So the logic must also be the logic of natural language and other forms of communication such as music and even human emotions like love. Natural language combines both strands of reasoning: the dialectic which convinces the mind and the other which appeals to the passions. The latter was widely studied as rhetoric in the context of oral human communication during the classical period ¹ and during the middle ages but has received scant formal attention. There are traces in utilitarianism and economics. Nevertheless there are wide areas of application today like in the logic of advertising. The logic of consciousness, perhaps the most difficult of all concepts, involves both strands. The 'qualia' belong to the rhetorical.

The different branches of logic have been advanced mainly with their own local assumptions, assumptions which are often not valid in these modern areas of application with topics like non-locality and contextuality, as in quantum mechanics and relativity. These introduced in the twentieth century are now essential components of applied logic.

Quantum mechanics ² means non-locality and relativity means contextuality. In the special theory of relativity truth values depend on dynamic context: for example time depends (objectively) on the inertial frame of the observer. Objective truth depends on user context. So truth is user-dependent. In the general theory of relativity truth depends on the context. Objective truth depends on the contents of the user context. This is the point that the world has no existence devoid of its content. It is not a box or a container for the contents. The contents make the container. Consequently many earlier concepts of mathematical space

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 $^{^1\}mathrm{Aristotle's}$ began his Rhetoric with the observation that rhetoric is the counterpart of dialectic.

 $^{^{2}}$ a distinction is needed between quantum logic (a model based on the non-distributive orthocomplemented modular lattice) and the logical reality of quantum theory (satisfying the correspondence principle) [16].

are inadequate for applicable logic. While these phenomena of logic may only prevail at low physical dimensions in quantum theory and at extreme distances and speeds in relativity, in the ultra-physics of the humanities they are part of normal everyday experience.

Formal logic grounded firmly in syntax has been extended to include logical semantics but very little is available for pragmatics. That is how logic is applicable to goings-on in the real world. Modern problems tend to be global and solutions therefore need to be integrable. All have to fit into the same scheme. Piecemeal they can be but views of one logic. The aim therefore is to identify that one logic which can encompass all the others. This is a natural logic.

2 Principal Universals and Universal Principles

What does logical monism require? First it can have no bounds and therefore must at least in principle be demonstrable to deal with all knowledge even if we are not clever enough yet to work out all the details. We need to examine each type of logic and to satisfy mathematical reasoning.

The post-modern world requires quite an elaborate definition of 'universal' incorporating the new concepts from the twentieth century. 'Universal' is a term of intension, its corresponding (but contravariant) extensional counterpart is 'global'. It is the extensionality of globalisation which is such an important feature of today's world. World War II was universal in that it was everywhere; the war against terror is global, that is <u>anywhere</u> rather than everywhere. Note that a universal of locals is not the same as globalisation because the former lacks connectivity. It is the connectivity of non-locality that gives globalisation [17, 18]. In this paper we use the word universal in a generic sense to include both intension and extension unless it is a context in apposition to global.

The notion of a universal logic has been around for some time. It may be implicit in the concept of Aristotelian syllogism as developed in his Organon [20, 26, 31]. Kant even claims Aristotle's logic as 'completed and perfect'³. The universal nature of formal logic was perhaps first made explicit by Raymond Lull (1232-1315/6) the Catalan philosopher, mystic and martyr from the Balearic Islands. He believed in the unity of philosophy, law, medicine and theology. He advocated a Nova Logica for application in the science of law and medicine ⁴, promoting experience and experiment over authority ([32] page 352). He claimed that the science of law 'being very prolix and diffuse' must be reduced to syllogisms and produced four works on the subject of ars juris. In the mediaeval period he was considered along with Adam and Solomon as one of the three wise

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 $^{^{3}}geschlossen und vollendet$ in the preface of his second edition to Critique of Pure Reason (Kant, I, Kritik der reinen Vernunft 2. Auflage 1787, Bviii, reprinted in [22]).

⁴Liber de modo applicandi novam logicam ad scientiam juris et medicinae referred to in Pascal's biography Vindiciae lullianae (ii page 104) cited at page 311 footnote 1 of [32].

men of all time 5 ([32] at p.402).

Since at least the time of Raymond Lull there has been a continuing aspiration for a fundamental language of reasoning that could satisfy all problems. This was the motivation for Leibniz to propose his principle of *calculemus*. The quest for a universal language in the period from Lull to Leibniz has been thoroughly studied by Paolo Rossi [35]. Before 1600 the ninth Earl of Northumberland in advice for his son's education was recommending a 'vniuersall Grammer' which was 'sutche a doctrine generall as discouereth ... the best wayes to signify the conceipts of our minds' ⁶. Lull's method was geometric and his extensive figures have been described by Hegel as thinking machine. Leibniz described his own proposal as an 'alphabet of human thought'. Both Lull and Leibniz were motivated by practical applications of their day. In those times the issues were often generated from religion and theology. This well illustrates the universality of logic whatever the applications of the day. Lull believed that the logic of Christianity was conclusive and could therefore be used to convert Jews and Islam to Christianity.

Leibniz living in the days of the reformation was troubled by the conflicting claims and counter-claims of Roman Catholicism and Protestantism and believed that a formal method was available to resolve the issues and determine the absolute truth. Both Lull and Leibniz envisaged their method as a practical calculating machine and produced diagrams of logic which could be used for practical inferences.

In 1854 the clergyman George Boole published his "An investigation into the Laws of Thought on Which are founded the Mathematical Theories of Logic and Probabilities" to analyse laws of diet. Boole was able to apply the language of algebra as developed over the previous 200 years to reduce the syllogisms of Aristotle to a simple algebra and so effectively merging logic with mathematics.

3 Modern Approaches

Around the same time as Boole, Arthur Cayley the barrister-at-law of Lincoln's Inn and Cambridge mathematician was advancing ideas about universal transformations that led to the development of group theory and eventually category theory. The twentieth century saw the emergence of unifying schools of pure mathematics like Bourbaki who claimed the three components of universal unity as algebra, topology and order. The underlying unification was first-order predicate logic as expressed in first-order Zermelo-Frankel set theory ⁷. Thereby "officially" asserts Buchberger, "there is one uniform logic system in use" ([4] at p.195). However Buchberger criticises what he calls "Bourbakism" as a system

⁶Stephen Clucas in his translator's introduction to [35] at p. vii.

 7 with or without the axiom of choice -- a critical feature in applied logic systems.

⁵Tres sabios hubo en el mundo, Adán, Salomón y Raymundo.

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to integrate the power of logic by identifying a number of gaps ⁸ because it would treat algorithms as sets. That is it would treat as the same, different algorithms that have identical input/output. Buchberger advocates replacement by the 'symbolic computational' power of computer algebra with systems like Mathematica TM. Not being concerned with applied systems however, Buchberger does not account for the Church-Turing computational gap [17, 18] which seems the 'make or break' point of any realisable system of universal logic.

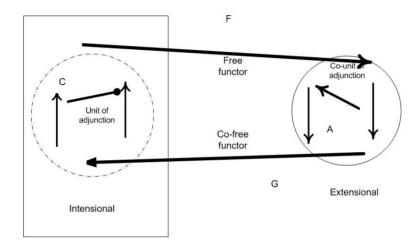


Figure 1: Adjointness between intensional and extensional

Buchberger typifies the class of logician who approaches the question of universal logic 'bottom-up'. Other examples are fibring logics [13] particularly cryptofibring [5] and other proposals [33, 34] for unifying existing logics. In addition to overall philosophical objections to assumptions in a universal logic as discussed above, these methods also involve extensive particular assumptions ⁹ and the collapsing problem. The technical deficiencies arise because generally these methods follow a Kripkesque path. Kripke semantics is set semantics. The collapsing problem is an example in fibring where any logic collapses to a Boolean logic. This arises from applying the axiom of choice [7]. Axiomatic set theory is usually considered from a background of proof theory and model theory to be a consistent system [2]. This may well be sufficient from the viewpoint of current pure mathematics because of its great sustained successes over wide areas

⁸Logic gap, system gap, syntax gap, mathematics gap and prover gap ([4] at p.195-198)

⁹Gabbay characterises the logical systems for general fibring with various assumptions under eight headings: syntactical assumption, semantical assumption, the evaluation function, definition of the fibred language, the fibring function, simplifying the fibring function, general definition of fibring and the notion of dovetailing. For instance it is to be assumed that the fibring function can provide acceptable values of one logical system in another but this still has to be hand-crafted. "In general the nature of the allowed fibring function F has to be worked out for each application with possibly some correctness theorems involved" ([13] at p.12).

[27, 28]. However it is not universally consistent without presupposing some kind of Platonism ¹⁰. Deficiencies in ZFC are now being taken more seriously. See[21]. In applicable mathematics the point at which a system fails is much more critical. Set theory fails without doubt quite dramatically in most applications in the humanities. It is very obvious in any context involving natural language and this includes most modern information systems. Where potential failure is less clear but perhaps still more critical is in applications of biology and medicine. Certainly the sixteenth problem of Hilbert's Programme to show the axiomatic basis of physics has failed in this objective [6]. While it may be often possible to plug a gap in a particular example with a hand-crafted solution this cannot be the approach of a universal logic.

A pervasive difficulty of the bottom-up approach is in deciding on a definition for equivalences of proof [8]. From a top-down view however, the topic of isomorphism and equivalence is an inherent property of the system ¹¹.

4 Arrows not Sets

The top-down approach is natural in category theory which is based on the arrow rather than the set. Philosophical closure is possible at the top: the universe is an arrow in its formal sense as used in category theory. This gives the formal arrow an existence for any constructive proof as found in applied categories. The advantage as a basis to express universal logic is that no assumptions ¹² are needed. This means that the concepts like natural numbers and negation as well as all the paraphernalia with these are not available to us. However this is to be expected. For these concepts it will be noted for universal logic do not exhibit the characteristics of non-locality nor are they relativistic.

As one constraint the arrow is cartesian. This is the extensional form of the intensional principle (or assumption) that the universe exists. The extensional is given by the existence of real entities and the structural inter-connectivity anywhere, that is a cartesian-closed category that is also locally cartesian closed. The logical power of category theory was noted by early categorists [29, 11, 15, 23]. This was probably the first systematic introduction of geometric logic into mathematics corresponding to the earlier algebraic introduction by Boole ¹³. They were able to show how the whole of classical logic could be exploited and enriched in formulations expressed by the arrow. Nevertheless their approach is heavily reliant on the category of sets and therefore subject to the same limitations as

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 $^{^{10}}$ as admitted by Gődel in his 1933 lecture 'The present situation in the foundations of mathematics' cited with approval by Feferman in 1992. See 'Why a little bit goes a long way: logical foundations of scientifically applicable mathematics', reproduced in [10].

¹¹and straightforward to apply to methods and models [19].

 $^{^{12}}$ or other assumptions if the reader wants to insist that the existence of the universe is itself a philosophical assumption!

 $^{^{13}\}mathrm{Aristotle}$ employed geometric logic as in the discussion of distributive and corrective justice in book v of his Ethics.

noted above for first-order predicate logic.

4.1 Adjointness

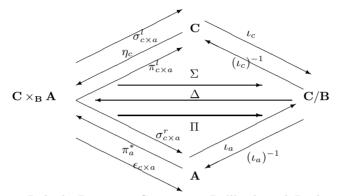


Figure 2: Dolittle Diagram: Composing Pullback and Pushout of extensional along intensional

Although well known in many guises it was only perhaps in the 1970s that the full universal significance of adjointness became apparent. In his seminal paper [24] has shown that intensional and extensional forms are related as in Figure 1 by adjoint functors between categories¹⁴ C and A of opposite variance. The free functor F provides the modal basis of possibility, the co-free functor G the modal necessity and normative and other deontic concepts. The unit η and counit ϵ of adjunction are defined by $F \dashv G, \eta : 1_C \longrightarrow GF, \epsilon : FG \longrightarrow$ 1_A . Figure 2 is a Dolittle diagram which is convenient in applied categories to combine a pullback and a pushout. The Dolittle diagram is able to subsume algebra, geometry and the openness of topology as well as adjoint ordering. These include the three principles of the Bourbaki School which is therefore merged with category theory.

This Figure 2 diagram shows adjunctions between all the expected components. The arrows are to be interpreted in the context of either the pullback or the pushout as appropriate. It also shows the generalised existential qualifier (Σ) and the generalised universal quantifier (II) in the Stone duality between limits in the pullback and colimits in the pushout. The stability functor (Δ) is respectively right-and left-adjoint to these: $\Sigma \dashv \Delta \dashv \Pi$. The unit of adjunction η is the ontological measure of the dialectical logic while the co-unit ϵ is the phenomenological measure of the rhetorical logic. A critical feature is that for any given category

 $^{^{14}}$ In applied categories there is not much use for small or concrete categories and therefore an ordinary rather than a gothic font is used for the name of an applicable category. Pure categorists would call these enriched categories or a class and use gothic script to label them.

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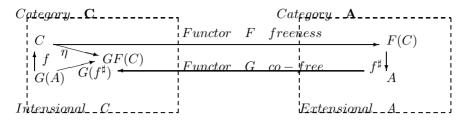


Figure 3: Proof of Dialectic Logic as Natural and Unique

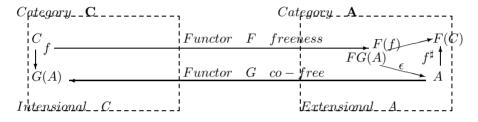


Figure 4: Proof of Rhetoric Logic as Natural and Unique

C an arbitrary functor F, both category A and functor G are uniquely defined. That is they are natural and the definition arises from the composition of the adjunction triangles in Figures 3 and 4. The logical structure of the adjunction is shown in the corresponding pullback of Figure 2 where some (sub)category (B)is the conclusion of a material implication $(A \Rightarrow B)^{-15}$. The category C/B is a slice category [14] ¹⁶. With no *a priori* assumptions so far, the resulting natural internal structure is that of a topos, a negation-less Heyting logic. This reduces to the familiar Heyting and Boolean logics when the appropriate assumptions are made ¹⁷. This is then the natural logic ¹⁸ of the universe for the simple reason it is what we end up with when we make no assumptions other than to take the universe as an arrow. How this view of universal logic copes with the common problem of negation can be well seen from an application given below. The adjunction of Figure 1 induces a monad $< T : GF, \eta : 1_C \longrightarrow GF, \mu : T^2 \longrightarrow T >$.

 ^{17}Ex falso sequitur quod libet for Heyting and Tertium non datur for Boolean.

 $^{^{15}}$ Even if C and A are the same category the pullback is not symmetric. There is a parity distinction between the left and right projection of the limit, first noted by Lambek according to ([8] at p.483). More specifically the left projection has female characteristics and the right projection male [25]

 $^{^{16}}$ It is the stability condition between slice categories of variable *B* that satisfy non-locality, connectivity from the definition as locally cartesian closed. The slice *B* is a variable for space and time. As a preorder *B* expresses space-time; as a partial order it will deal with more conventional time scales like past, present and future with a capability to express these with precision even distinguishing classical types of time e.g. universal time, sideral time, etc.

¹⁸Logics of the same flavour are the paraconsistent logic [3] or minimal logic [1].

Identity and associativity are given by the commutative diagrams in Figure 5. This monad is the fundamental unity and associativity of logic in the universe. It is perhaps therefore no surprise that it can be identified with Leibniz' concept of the monad ([17] at p.308).

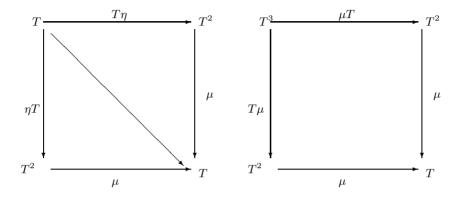


Figure 5: Identity and Associativity of the Mondad-induced by a Standard Adjunction

5 Demonstration

A test sentence used in the literature for comparative logics is:

John said that Mary believed he did not love her

From the perspective of this paper the test sentence has some deficiencies. It is not a sentence taken from the real world but artificially constructed. It is not a well-formed formula (ie a wff generated by natural language) because it lacks context. For the context it matters to whom John is speaking. Even a sentence from a novel would not completely satisfy the true context test because a novel is not an open system like the real world. A sentence needs to be taken, say, from some news item but even then would only be a wff in its immediacy. However, this sentence does have some value in the application of objective scientific method for us because it is not biased or prejudiced in that it has been invented by us ¹⁹.

Category theory has both the rigour and flexibility of natural language. Without a context the default would be that John made this statement to himself. That is there is an intensional adjunct between John and himself: $J \longrightarrow J$ where the arrow is a left and right adjoint pair $F \dashv G$. This naturally expresses a

 $^{^{19}}$ Another basic inadequacy is that the sentence does not include modal considerations even though it includes the word 'believe' for a belief is an expression of fact not contingency.

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kind of reflecting and so includes the notion of parity mentioned in the footnote above. The time in the past when the statement was made would be taken care of by the appropriate time slice of B as also already mentioned. The extension of John's reflection is Mary's belief which is again a reflection of her own and expressed by the adjunction $M \longrightarrow M$ where the arrow is another left and right adjoint pair $F' \dashv G'$. The extension of this adjunction is that John does not love Mary. The arrow representation is $J \longrightarrow M$ where the arrow is a left and right adjoint pair $F'' \dashv G''$. Here we see the negation and also the phenomenological aspects come into play. The quality of the love that Mary has in mind will be expressed by some value of ϵ in the partial order $\top \longrightarrow \bot$. If it is Mary's belief that John is absolutely loveless towards her the value of η will be \bot in the partial order $\bot \longrightarrow \top$. That is his love is 'rock-bottom'. In a Boolean world the unit collapses to a negative operator as a definition of $\eta \longrightarrow \bot$ while any qualia of the 'loving' are probably lost. This shows the sophisticated characteristics of natural language which often paraphrases negation without using the word 'not'.

It is to be noted from the identity diagram for the monad induced by this adjunction that the unit η commutes across a T : FG pair. So ηT when composed with μ is equivalent to $T\eta$. This is the difference in the natural language versions between 'Mary believes that John does not love her' and 'Mary does not believe that John loves her'. Note this only applies in a context of $\mu : T^2 \longrightarrow T$ which specifies a particular circumstance. The logic of each subclause in this sentence is an adjunction. It is a chain of predicates from intension to extension.

The composite sentence is given in Figure 6. Composition of three primed adjunctions $\langle F', G', \eta', \epsilon' \rangle$, $\langle F'', G'', \eta'', \epsilon'' \rangle$, $\langle F''', G''', \eta''', \epsilon''' \rangle$, make a triple adjunction $\langle F, G, \eta, \epsilon \rangle$ expressing the ontological and phenomenological features of the sentence as a whole: John said that Mary believed he did not love her.

The unit and counit of adjunction of the whole sentence may be determined as follows:

1.

- 2. The unit is a composition of $\eta: 1 \longrightarrow G'F'$ with $G'\eta''F': G'F' \longrightarrow G'G''F''F'$ with $G'G''\eta'''F''F': G'G''F''F' \longrightarrow G'G''G'''F'''F''F'$
- 3. The counit is a composition of $F'''F''\epsilon'G''G''': F'''F''F'G'G''G''' \longrightarrow F'''F''G''G'''$ with $F'''\epsilon''G''': F'''F''G''G''' \longrightarrow F'''G'''$ with $\epsilon''': F'''G''' \longrightarrow 1$

The general algorithm to calculate $\langle F, G, \eta, \epsilon \rangle$ is given by $\langle F'''F''F', G'G''G''', G'G''\eta''F''F' \bullet G'\eta''F \bullet \eta', \epsilon''' \bullet F'''\epsilon''G''' \bullet F'''F''\epsilon G''G''' \rangle$

Fuller workings of this general algorithm and justification of its generality can be found in [36].

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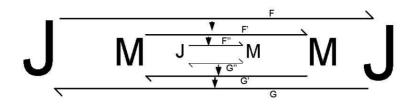


Figure 6: 'John said that Mary believed he did not love her'

6 Concluding Remarks

As Béziau has pointed out we are no longer living in a classical world but a postmodern one so we should expect a postmodern logic. Distinct points where the postmodern departs from the approach of classical or modern logics have been mentioned in passing: differences between universal and global; the non-locality of space and time; the shedding of pre-conceived structures and assumptions; and balancing the dialectic with the rhetoric. The asymmetry in the pullback may even suggest that feminist writers like Andrea Nye may well be justified in attacking the likes of Frege for patriarchal prejudices of modern logic [30] in view of the unjustified concentration on dialectic at the expense of the rhetoric. Figures 3 and 4 show that it is a matter of perspective across levels, and not oriented at just one level. The dialectic appears ostensibly at the intensional level and the rhetoric at the extensional but the unit and counit values are derived from a comparison of the levels. By unravelling the cross-linkage we may be deconstructing 'meaning' in the sense of Derrida for meaning is the intrinsic significance of the intension/extension relationship. The arrow is perhaps then a footprint of the unfathomable (as described by Derrida) culminating in the double arrow of the adjunction as the one logic of this postmodern age.

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