

What is Law?: the perception of Category Theory

Nick Rossiter

Northumbria University, UK

nick.rossiter1@btinternet.com

Michael Heather

University of Cambridge, UK

Trinity College

michael.heather@trinity.cantab.net

<http://nickrossiter.org.uk/process>

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Law and Logic

- Law controls our lives.
 - We live under the rule of Law.
 - also subject to the laws of science.
 - Law in one way or another affects every facet of human life.
- We need therefore to understand better the way that Law operates.
- Do we even know the nature of law?
- To understand it we need to examine its roots.
 - For they lie in logic.

Scientific and Human Law

- There is *prima facie* a distinction between the scientific law and human civil and criminal laws in that the former are always strictly obeyed whereas the latter may not be observed.
- However delving deeper we find that scientific law is not always universally obeyed.
- Law whether scientific or human operates from a higher level.

Metaphysics and Law

- Current mainstream mathematics that derives from set theory cannot deal directly with separate levels
 - but only collapses them into a model of first order.
- Alfred North Whitehead advocated a move to Process at the higher level of metaphysics.
 - Unfortunately Whitehead did not have a formal metaphysical language to replace the mathematics of set theory.

Law and Category Theory

- Today Category Theory now fills that gap.
 - It is a formal language that operates across four levels recursively
 - features like adjointness between universal limits and co-limits that were not appreciated until the 1970's.
- All Laws arise from this natural adjointness as underlying functors.

Applied Category Theory

- There is the caveat that the current representation of Category Theory only holds up to the natural isomorphism of the axioms of set theory.
- Applied Category Theory needs to hold up to the natural isomorphism of Physics.

Logic and Truth in this Postmodern Age 1

- Law is logic
 - but logic in the context of human society at a specific place and time.
- Thus Roman Law provided detailed regulation for a city state relying on slavery
 - while in sharp contrast the positive norms of the last two centuries provide for modern nation states but both are congruent and essentially local in extent.

Logic and Truth in this Postmodern Age 2

- Now globalisation ushers in a new postmodern age which
 - extends our understanding of Logic such that even Truth itself may not always be true.
- We are being driven to higher forms beyond first order positive Comtean law.

Logic and Truth in this Postmodern Age 3

- The harbinger of postmodern philosophy, Alfred North Whitehead (1861-1947), points us
 - back to Nature
 - forward to the metaphysics of Process.
- Law is relational held together by natural intuitionistic logic.
- The global whole consists of many local interacting dynamic parts that control human destiny.
- These may be understood formally within the adjoint functors of Topos Theory

Law & Logic in Metaphysics

- ❖ Newton advanced Physics in first order predicate logic as model of reality with excluded middle
- ❖ Contemporary Law needs Metaphysics as higher level Physics of reality in impredicative logic with included middle
- ❖ Heyting Logic as the internal language of a *Topos* in Categories
- ❖ What is postmodern truth?
- ❖ All truth is partial truth!

Inference and
causation in
classical, modern
and post modern
logics

Language	Logic	Implication Causation
Classical Set Theory	Boolean $p \rightarrow q$	'p implies q' defines (not p OR q) in a closed world where p and q may be independent
Modern Topology	statistical/ probabilistic inference e.g.:Bayesian, fuzzy set, paraconsistent, Deontic logics	Undecidable or uncertain consequences in an open world
Postmodern Metaphysical (i.e. not a reductionist <i>EML</i> model) Topos Theory	Heyting $\frac{C \leq A \Rightarrow B}{C \wedge A \leq B}$	A in context C implies B in context C: defines $A \Rightarrow B$ (exact/co-exact limits)

Historiosophy of the Metaphysics for Logic in Law

Year	Age	Law	Logic
BCE to 1840	Classical theistic	Natural Law	Inherent Logic of Natural Language [including legal fictions at Common Law]
1840 to 2000	Modern Secular atheistic/agnostic	Positive Law	Boolean Binary world with excluded middle Simple inference and causation when conditional <i>if and only if</i> holds
2000 onwards	Contemporary Post-Secular Spiritual wellbeing	Postmodern Law	Heyting world with included middle Higher level complex inference and causation <i>A</i> implies/causes <i>B</i> in a context <i>C</i> if & only if both <i>C</i> & <i>A</i> are in the context of <i>B</i>