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The Physicalism of Category Theory

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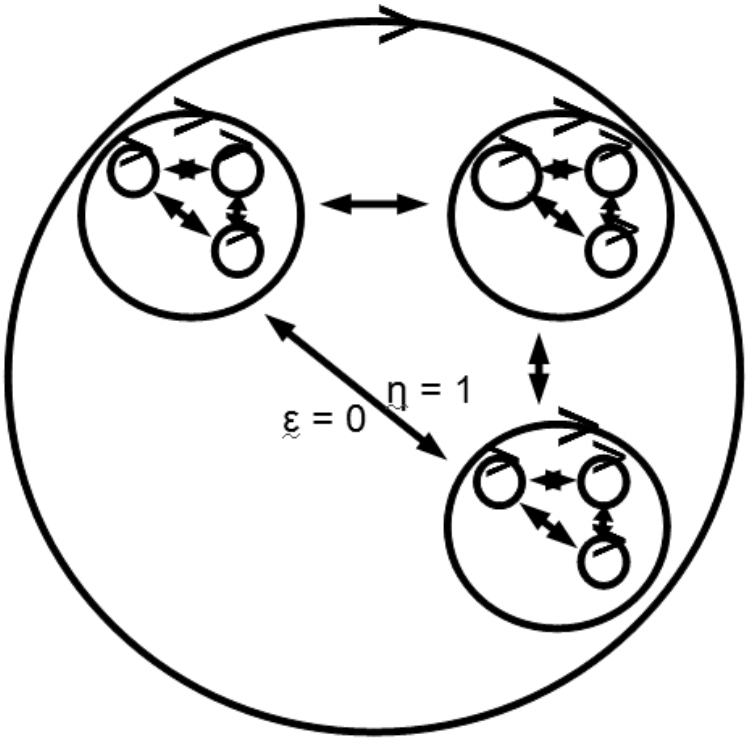
Civilization progresses with advances in science, supported by applicable mathematics. For three millennia the mathematics consisted principally of arithmetic and geometry. For the last 3 centuries these have been joined by algebra and the calculus to accelerate progress particularly in engineering and technology and leading to new sub-branches of applied mathematics like fluid mechanics and thermodynamics. The mid 20th century saw the addition of this new branch of mathematics, category theory, which has rather exceptionally in the history of science developed entirely within the confines of pure mathematics rather than being driven by practical needs.

The origins in pure rather than applied is significant. Category theory is not limited by any preconceptions of the real world but operates above reality at the higher level of metaphysics and not as a model at the lower level. Metaphysics brings with it new features for applicable mathematics with a vast potential yet to be fully explored. A profound advance is the great increase in formal power. The upgrade from modelling to metaphysics also provides wider scope for the use of mathematics in biological and human sciences in particular. Finitary mathematics by virtue of Gödel's theorems is limited to relying on the closed world assumption and limited interpretive capabilities of first order predicate logic. Not only is category theory free of these constraints but with an interpretation of process physics it also integrates the local into the global as needed in many of today's world problems. Intension is the ultimate reality with the extensions as possible physical interpretations.

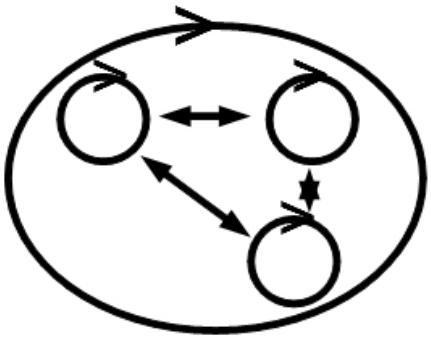
Existence in physics differs from existence in mathematics. Mathematical objects require no more than logical consistency to exist. Physical objects only exist if they are observable. Extensions are perceptions in this sense. Physicalism questions any existence that is not physical. Category theory recognises the existence of higher level types along a path of recursive transformations allowing a smooth progression from elementary particles through process relations for notions such as human emotions or the workings of the mind up to concepts like consciousness all within the physical world.

By recursive process, category theory even provides the theory of its own interpretation by means of adjoint contravariant functors between the intension of metaphysical reality and the appearances in extensional forms.

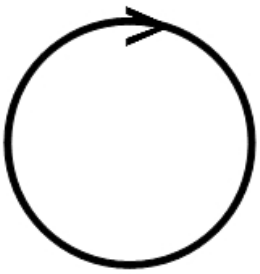
Identity Natural Transformation
($\underset{\sim}{\eta} \rightarrow 1$, $\underline{0} \rightarrow \underset{\sim}{\varepsilon}$)
[Large Product Category]
[Boolean, extensionally closed]
[Locally Cartesian Closed]
[Power Set]



Identity Functor
(1_F, $\underline{O}_G$)
Product Category C^{OP} X C
Category of Sets



Identity Arrow
($\mathbf{1}_A$, $\underline{O}_A$)
[object]



[TOPOS]
Intension: identity natural transformation, closed
Extension: open
(T, η, μ) (S, ε δ)
[Heyting, extensionally open]
[Cartesian closed category]
[Double Power Set]