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NEW FOUNDATIONS FOR APPLIED MATHEMATICS WITH CATEGORY THEORY

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Abstract:

Quine's 'new foundations' for set theory might serve pure mathematics but now seem inadequate for a mathematics applicable to our current open interactive global world comprised of many different possible logical views and meanings. For the closed world assumption (CWA) only holds locally. Category theory shows diffierent ontologies and semantics to be examples of exact and coexact adjointness related by pairs of contravariant functors.

Aristotle's first and second intentions were resolved in 1662 by Port Royal Logic into intension and extensions. The formal approach of Frege's predicate logic in 1879 represented meaning as arithmetic but on account of the CWA is not universal. Finitary mathematics was rigorously expounded in Whitehead & Russell's (1910) *principia mathematica* but gave rise to paradox and was repudiated explicitly by the latter and implicitly by the conduct of the former in his move to the philosophy of process. Nevertheless their axioms with the CWA was the starting point for Gödel's doctoral thesis (1930) that first order (boolean) semantics is complete, his metamathematical 5th theorem (1930) that predicates are realizable as coded by number and Gödel's 6th theorem (1930) that axiomatic systems of number/sets are undecidable. Hilbert's finitary mathematics programme for the *Entscheidungsproblem* was therefore undecidable. Put into the language of scientific method this means a mathematical model can only be validated by experiment to first order. That is measurement only holds locally.

This is the context of the Church conjecture (1932) for the effective calculability of number by recursive functions that has led to the Church-Turing Thesis (1936) and the effective computability of number. However any computer restricted to a von Neumann architecture is a calculator and with the CWA not a general logic machine. The same limitation applies to the Church-Turing-Deutsch Principle (1985) for effective quantum computability in parallel. A real quantum computer needs a full quantum theory beyond the first order model with logic gates beyond the CWA.

Wigner's 'unreasonable effectiveness of mathematics in the natural science' is limited to first order problems by the CWA. Most problems of the 21st century arise in open systems in science A formal representation of our current open interactive global world is to be found in the topos of category theory where the CWA is replaced by the (formally defined) natural open logic of the preorder. However this like all mathematics needs to be overpined by a metaphysical philosophy. Whitehead's later theory of the Universe as a non-stationary process appears at present to be the only one that can escape the clutches of Gödel's undecidability to address the interaction of the local with the global.